Study of the Properties of Nuclear Energy Levels In Deformed Even-Even Medium and Heavy Mass Nuclei

THESIS

SUBMITTED TO ALIGARH MUSLIM UNIVERSITY, ALIGARH
IN PARTIAL FULFILMENT FOR THE
AWARD OF THE DEGREE OF

DOCTOR OF PHILOSOPHY

in

PHYSICS

by

VIR PAL VARSHNEY
M. Phil.

DEPARTMENT OF PHYSICS
ALIGARH MUSLIM UNIVERSITY, ALIGARH
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A B S T R A C T

A systematic study of the nuclear properties like energy, mean life and B(E2) values of the excited levels of even-even deformed nuclei has been done in the frame-work of Asymmetric Rotor Model (ARM) of Davydov-Filippov and Davydov-Rostovsky. A large number of medium mass nuclei for which non-axiality parameter (γ) lies between 15^{o} and 25^{o} have also been studied along with heavy mass nuclei to reflect the characteristics of ARM. We have used, for the first time, the ARM dependent intrinsic quadrupole moment (Q_o) to calculate the deformation parameter (β) to keep consistency withe the theoretical predictions. The entire thesis has been divided into six chapters.

In the first chapter the revival of the interest in ARM together with its importance in describing the observed rotational and vibrational spectra of medium and heavy-mass nuclei have been discussed. A schematic picture of collective modes of excitations of non-spherical even-even nuclei and the basic physics involved therein have been described. A comprehensive survey of the work reported in the literature has been done. The utility of the present work for future researchers has been mentioned.

The second chapter deals with the review of various theoretical approaches. In the review of theoretical approaches different nuclear models used time to time in

describing the collective excitations of deformed even nuclei have been presented. Three models i.e. Bohr-Mottelson, Davydov-Filippov and Davydov-Rostovsky have been discussed in detail to obtain certain mathematical expressions for reduced electric quadrupole transition probabilities for inter and intra band transitions in deformed even nuclei. Other approaches like variation of angular momentum projection approach, pairing plus quadrupole model, pseudo SU(3) model, Sahu's approach, Microscopic asymmetric approach, Toyama's approach, boson expansion model and new rigid asymmetric rotor model which are used for comparison of results, have been discussed in brief.

In third chapter, the effect of spin of nuclear shape has been studied. Davydov-Filippov calculations of reduced electric quadrupole transition probabilities for $4^+ \rightarrow 2^+$ and $6^+ \rightarrow 4^+$ transitions, which have not been done so far, have been presented for the first time for medium as well as heavy mass nuclei especially in the region $15^\circ < \gamma < 25^\circ$. In this region there is no solid theoretical argument against the rigid shape of nuclei. Systematics have been obtained on plotting the factor F $[B(E2)_{exp}/B(E2)_{theo}]$ versus γ in DF model for $2^+ \rightarrow 0^+$ and $4^+ \rightarrow 2^+$ transitions and in DR model for $6^+ \rightarrow 4^+$ transition. It is inferred that the assumption of rigid triaxial shape with fixed shape parameters β and γ is an excellent approximation to the actual wave functions only upto spin 4 and thereafter the nucleus begins to get rid of rigid shape and acquires β-and γ-vibrations. Our study lends support to the triaxial shape of Davydov-Rostovsky and suggests a limit of I=6 beyond which the rigidity of nucleus is unacceptable. This conclusion definitely contradicts the Turner et al.s' view point of abrupt phase transition of the rotational mode from axially symmetric to asymmetric shape at I = 8.

The fourth chapter deals with the Samarium isotopes which have been a chalanging theoretical problem and cover the entire region from nearly spherical to well deformed. This study has been done in the frame work of rigid rotor model. The results thus obtained have been compared with the various existing models. ^{148}Sm nucleus is believed to be basically spherical, ^{154}Sm is thought to be well deformed while $^{150,152}Sm$ nuclei are transitional. Various approaches have been applied in past but none of them were found to be fully successful in explaining the known B(E2) values and B(E2) branching ratios of inter and intra band transitions. In this work a new method of determining γ is employed together with the existing methods. New empirical linear relationship has been observed in model dependent intrinsic quadrupole moment Q_o and non-axiality parameter γ. It has been inferred that mass number A and charge number Z also play an important role in assigning shape to the nucleus in terms of asymmetric parameter γ.

In the same chapter we have given the predicted mean-lives of rotational excited levels of Samarium isotopes. Our calculations assign 10^+ and 12^+ spins to known levels of 1.338 MeV and 1.817 MeV energies in case of ^{154}Sm, which were suspected to be 4^+. Our study also lends support for the evidence for triaxial shape in ^{150,152,154}Sm nuclei. From our calculations we have inferred that in ^{152}Sm nucleus the observed 2.158 MeV energy level should have 12^+ spin.

In the fifth chapter the earlier quoted results of systematic trends between the ratio F $[= B(E2: I+2 \rightarrow I)_{exp}/ B(E2: I+2 \rightarrow I)_{theo}]$ and non-axiality parameter γ have been modified by taking model dependent Q_o in heavy mass nuclei. Calculations were also done for medium mass nuclei. These modified systematic trends have been employed to predict the absolute B(E2) values and probable mean lives of rotational excited levels of even-even deformed nuclei upto $12^+ \rightarrow 10^+$ transitions. The predicted meanlives have been compared with their respective experimental values available in the literature and are found in excellent agreement in almost all cases. By employing Davydov-Rostovsky Model, the $K^\pi = 0^+$ vibrational band head energies ($E0^{+\prime}$) have also been predicted in some cases. Some of our predictions in respect of $E0^{+\prime}$ are found in very good agreement with their respective values given in Table of Isotopes. Our study supports the assumption of triaxial shape of Davydov et al. and confirms the experimentally observed $K^{\pi} = 0^{+}$ levels with energy of about 1 MeV as the K = 0 component of the quadrupole shape oscillations and interpreted as classical beta vibrations.

Chapter six describes the meanlives of 2^{+} excited levels of gamma vibrational band of even-even deformed nuclei in medium and heavy mass regions which have been predicted considering $2^{+\prime} \rightarrow 0^{+}$ and $2^{+\prime} \rightarrow 2^{+}$ transitions. The predicted values of meanlives of $2^{+\prime}$ state, calculated from $B(E2: 2^{+\prime} \rightarrow 0^{+})$ and $B(E2: 2^{+\prime} \rightarrow 2^{+})$ values are found similar. For the predictions of these meanlives the earlier reported B(E2) values were modified by taking model dependent Q_{o} and new systematics were drawn including large number of medium mass nuclei.

Using experimental known values of $B(E2: 2^{+} \rightarrow 0^{+})$, the $B(E2: 2^{+\prime} \rightarrow 0^{+})$ and $B(E2: 2^{+\prime} \rightarrow 2^{+})$ values have also been predicted for those nuclei for which the meanlives are not known. Our study supports the rigid rotor model and probable triaxial shape to medium and heavy mass region nuclei. This observation confirms $K^{\pi} = 2^{+}$ vibrations as collective in nature known as classical gamma vibrations.

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D E D I C A T I O N

TO

MY

FATHER

C E R T I F I C A T E

Certified that the work reported in this thesis is original work done by Mr. Vir Pal Varshney under my supervision.

(A.K. Chaubey)
Superviser

A C K N O W L E D G E M E N T

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Varshney

(VIR PAL VARSHNEY)

ABSTRACT

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CAPTION TO FIGURES

CAPTION TO TABLES

C O N T E N T S

CHAPTER - I

INTRODUCTION

The work reported in this thesis presents the systematic study of the properties of excited levels in the deformed even-even nuclei covering the medium as well as heavy mass regions in the framework of Asymmetric Rotor Model of Davydov et al.[1,2]. Although the assumption of rigid triaxial shapes with fixed shape parameters β and γ can be considered as an approximation to the actual nuclear wave functions, but this has been turned out to be very useful and is well supported by new data obtained from heavy ion experiments.

The revival of ARM[3-12] and also our interest in this work is based on the expectation that it may provide a reasonable phenomenological description of a nucleus in some domain of high angular momentum. The striking success of empirical relation[12] over the other existing approaches in describing the B(E2) values of gamma-ray cascades has contradicted the axial symmetry in the nucleus at lower spins. There is a remarkable point here that till now there is no sound theoretical argument against the triaxial shape of the nucleus around $\gamma = 20^{o}$, and the systematics of Gupta et al.[12] are of immense importance to experimental workers, since on using them the meanlives of rotational levels, absolute values of B(E2) and beta band head energies can be predicted. A knowledge of approximate values of meanlives etc is of great importance to the experimentalists. The study of B(E2) absolute values and B(E2) branching ratios for the transitions depopulating 2^+ state of the gamma vibrational band, done by Gupta et al. had confirmed low lying $K^\pi = 2^+$ levels as collective in nature, contradicting Zawischa et al.[13] view point. But they did not include the study of medium mass nuclei which have large γ values. Many nuclei of the new deformed region $28 < Z < 50$ and $50 < N < 82$ of medium mass having asymmetric parameter in the range $15^\circ < \gamma < 25^\circ$ have been added so that the characteristic of ARM could be reflected well. We have also employed for the first time the model dependent intrinsic quadrupole moment Q_0 in plotting our systematics which reflects the asymmetric spirit of the model in true sense.

Low-lying levels of the deformed even-even nuclei have been predicted by several phenomenological nuclear models[1,2,14,15]. Davydov and Filippov[1], using asymmetric rotor model (ARM) have derived the expressions for the reduced electric quadrupole transition probabilities from the first 2^+ excited state to the 0^+ ground state and from the second 2^+ excited state to the first 2^+ excited state, taking into account the interaction of rotation with gamma vibrations. Mc Gowan[16] compared $B(E2)_{DF}$ values with experimental ones and found a good agreement. However, somewhat limited agreement on the basis of the then available data had been observed by De Boer et al.[17].

It was suggested by Demille et al.[18] that the agreement may be improved if the Bohr-Mottelson vibration-rotation interaction and a centrifugal stretching correction, analogous to the type used in molecular spectra are introduced. The Davydov-Filippov model seems to be particularly useful for nuclei in the transition region, between rotational and near harmonic modes of collective excitation. Moore et al.[19] presented the tables which are useful for comparison with experimental energy levels of high spins, observed in heavy ions Coulomb excitations of rotational energy levels in even-even nuclei.

Van Patter[20] did a survey of available data for the gamma ray branching of the second 2^+ level for some even nuclei with $A > 30$. From those data, some ratios of reduced E2 transition probabilities were calculated and compared with the predictions of various theories. The comparison showed that the asymmetric rotor model of Davydov and Filippov was the most successful for predicting such ratios. The most striking success of the DF theory was the prediction of the ratio $B(E2: 2^{+\prime} \rightarrow 0^+)/B(E2: 2^+ \rightarrow 0^+)$ as a function of the energy ratio $E2^{+\prime}/E2^+$. However the branching ratio $B(E2: 2^{+\prime} \rightarrow 2^+/0^+)$ remained unstudied.

Tamura and Udagawa[21] have also plotted the ratio $B(E2: 2^{+'} \rightarrow 0^+)/B(E2: 2^+ \rightarrow 0^+)$ alongwith experimental values against $E2^{+'}/E2^+$ and found a very good agreement which became poorer in the extreme vibrational regions in the sense that DF did not explain the big fluctuations in the experimental values. The poorer agreement found above had been improved in their conventional theory. Such a result appeared to be quite natural because in both models the nucleus was considered to have a stable shape with a non-vanishing $\beta(=\beta_0)$ and $\gamma = 0$. The basic ideas of there two models were not the same since in DF the nuclear shape was considered to be fixed, while in conventional theory nucleus was allowed to change its shape. In vibrational regions, the difference becomes more apparent.

Davydov and Chaban[22] took into account the coupling between rotational and vibrational motions due to centrifugal forces and considered only β-vibrations. They assumed that the nucleus is rigid with respect to a change in equilibrium value of parameter γ, which determines the deviation of nuclear shape from axial symmetry. The $0^{+'}$ level which could not be predicted by DF model was explained alongwith other members of beta band in even-even nuclei.

Davydov and Rostovsky[2] modified the previous model by considering the shape of the nucleus to be changing as it passes into an excited state. A change of nuclear shape in excitations leads to some interaction of rotational motion with beta and gamma vibrations.

Qualitatively, the nuclei are deformed[23)] away from spherical shape because the nucleus is not the rigid structure and the nucleons outside the closed shells can set up tensions in the closed shell core, thereby establishing polarization of the nucleus. Fig. 1.1 illustrates the few nucleons outside the closed shell as orbiting around a spherically shaped closed shell core. If the forces between the external nucleons and the core are repulsive, there is a tendency to polarize the core by pushing the equatorial plane towards the centre of the nucleus to form a prolate spheroid. On the other hand if the forces are attractive, the polarization is accomplished by pulling out the equatorial plane to form an oblate spheroid. Experimentally the resulting deformation is observed as a quadrupole moment. Other observed rotational bands besides the ground state rotational bands in even-even nuclei can be explained by the hydrodynamic picture of the nucleus provided by the collective motion. These bands are formed from the coupling of the nuclear rotations to different vibrational oscillations as illustrated in Fig. 1.2. The simplest such oscillations deform the nuclei away from spheroidal shapes ($\gamma = 0$). Such type of oscillations can be described as a temporary deviation from axial symmetry and are called gamma (γ) vibrations. The symmetry around z-axis is no longer maintained during such vibrations. For gamma vibrations K

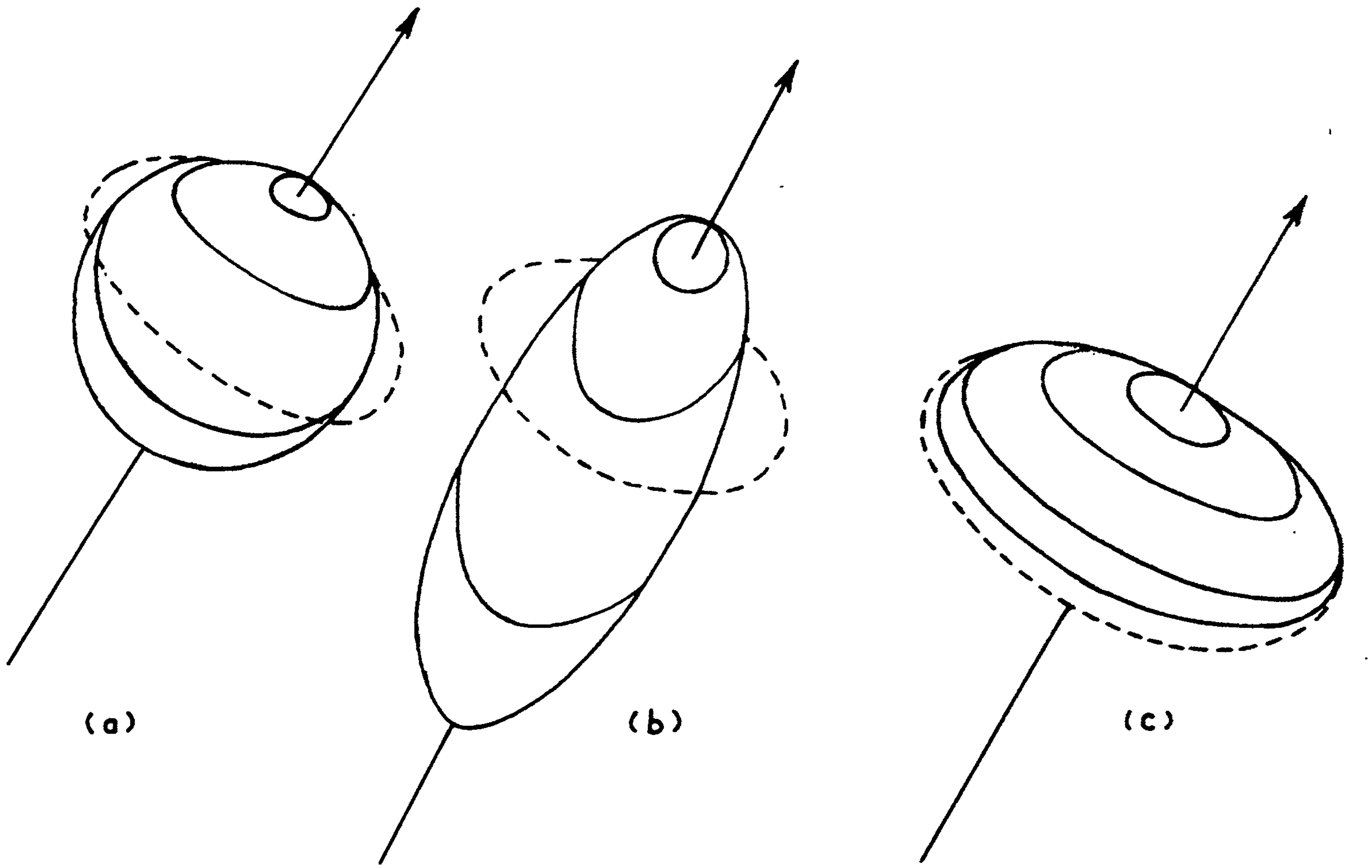

Fig. 1.1 The nucleons outside closed shells are believed to interact with the nuclear core in such a way that cause the core to deform from (a) Spherical into (b) prolate or (c) oblate spheroidal shape.

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experimental values, if known, and have found an excellent agreement for all nuclei except ^{134}Ba ^{134}Ce and ^{140}Ce It appears that the disagreement between the predictions of mean lives and their respective experimental values for the nuclei ^{134}Ba and ^{134}Ce is due to the neutron deficiency and consequently there is a significant influence of kinetic energy on the collective motion in the γ direction (Dobaczewski *et al* 1977) A special deformation dependence of kinetic energy can also privilege the deformation and make the wavefunction localise around it, which makes the deformation-dynamic. This is why the rigid triaxial model has failed to reproduce the experimental findings for these particular nuclei The experimental fact that the first excited 0^+ state ($E_{0^+} = 1760$ keV) is not well separated from the first excited 2^+ state ($E_{2^+} = 604$ keV) at higher energies for the nucleus ^{134}Ba also supports this viewpoint (Mayer ter Vehn 1975) The discrepancy in the results for ^{140}Ce is due to the fact that it has neutron number 82, which is magic

Our study lends support to the assumption of the triaxial shape by Davydov and Chaban (1960) and Davydov and Rostovsky (1964) and confirms the experimentally observed $K^\pi = 0^+$ levels with energy of about 1 MeV as the $K = 0$ components of the quadrupole shape oscillation, which are interpreted as classical beta vibrations in contrast to Zawischa *et al* (1978), who doubted the collective nature of low-lying levels and suggested that only high lying $K^\pi = 0^+$ resonances are classical beta vibrations

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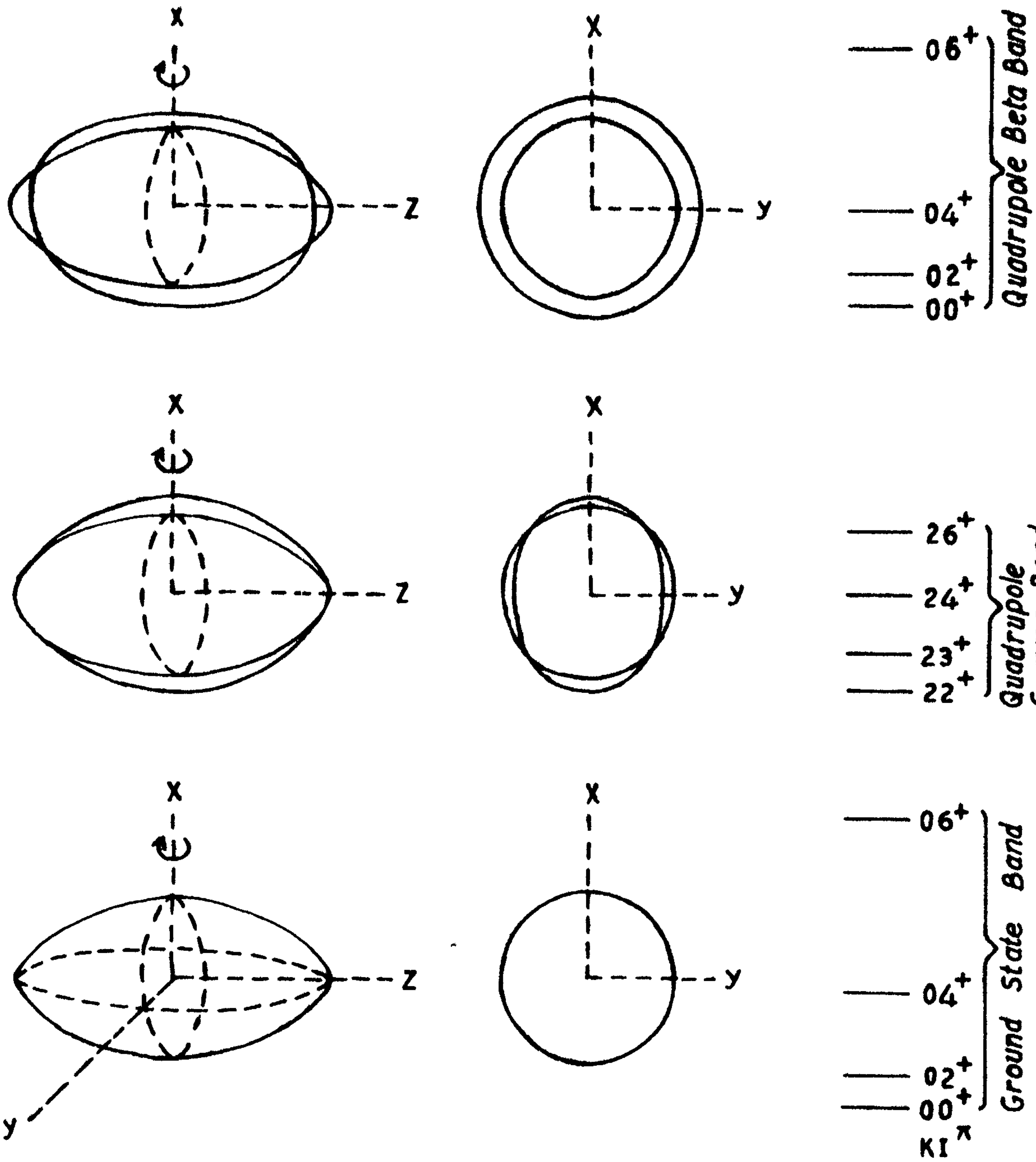

Fig. 1.2 Schematic picture of collective modes of excitation of non-spherical even-even nuclei.

has a value 2. Rotational states are then added to this state to give a set of levels for which I = K, K+1, K+2 etc where K is the projection of total angular momentum I, on the body fixed 3-axis. Quadrupole vibrational motion can also take place in a plane parallel to z-axis. Such motion produces deformation (β) hence are called beta (β) vibrations. In this, there is no change of shape of a cross-sectional cut through the equatorial plane of the nucleus, only an expansion and contraction of the circularly shaped nuclear surface takes place. For this set of rotational states K = 0 and the nucleus has at all times the same basic shape as the ground state, but with an added time dependent oscillatory change in the magnitude of its deformation parameter (β). The rotational states thus formed have spins as the ground state band i.e. I = 0,2,4 etc.

The observable rotational motion is possible, if the nucleus is pictured to be a fluid drop or to have any form with a definite surface. This rotational effect can be either rigid in which case particles actually move in circles around the axis of rotation or wavelike in which case particles perform oscillatory motions and only the geometrical shape of the drop changes. Such wavelike rotations are shown in Fig.1.3 and are observed only in deformed nuclei, because the apparent motion is solely a surface phenomenon. The observed moment of inertia for deformed nuclei are smaller than those for rigid

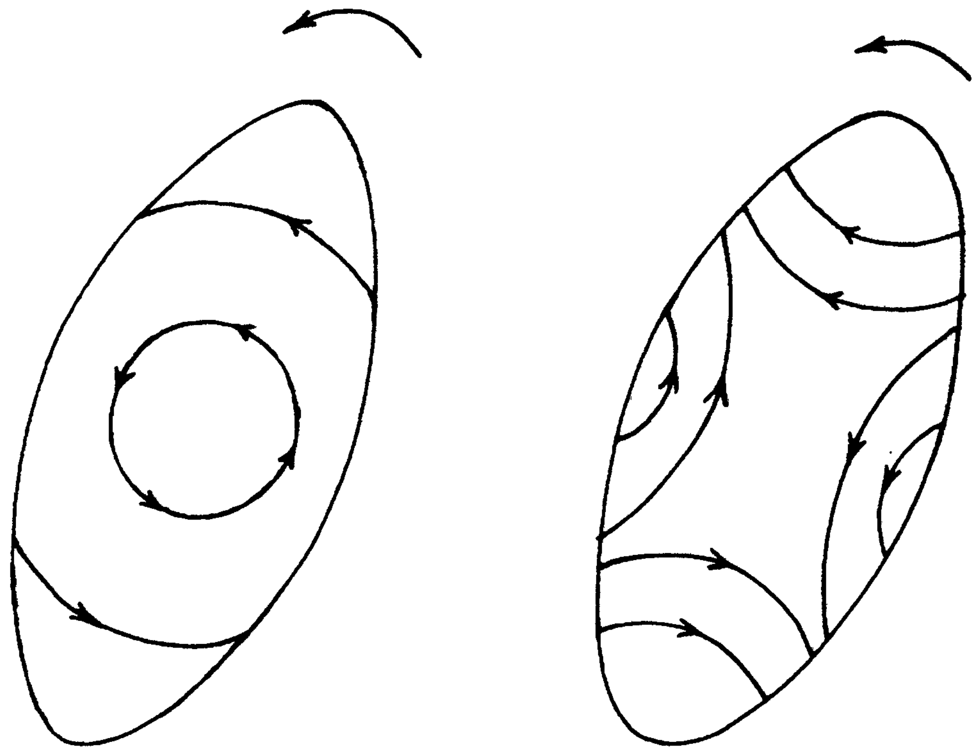

Fig. 1.3 Pictorial illustration of (a) rigid and (b) wave-like rotational motion of deformed nucleus.

rotors. They are, however, larger than expected for purely wavelike surface motion. Thus the apparent rotational motion is intermediate between rigid rotation and wavelike surface motion of constituent particles. According to the collective model, moment of inertia of nuclei can be determined from the energies of their rotational states. Rotational energy levels of an axially symmetric nucleus can be described by the three constants of motion: I, the total angular momentum: K, the projection of I on the nuclear symmetry axis: and M, the projection of I on a space fixed axis. For a wavelike rotation, there can be no rotation about the symmetry axis. The quantum number K is therefore a constant for each set of rotational levels and represents an intrinsic angular momentum for that band.

Davydov et al.[1,2] assumed the nuclear deformation parameter β and the non-axiality parameter γ to be permanent to some degree which were dynamic in Bohr-Mottelson model[24]. Further more they assumed deformations in both the elongation parameter β and the asymmetry parameter γ. Excitation of the lower energy levels is caused from rotation of the whole structure and the gamma-vibrations have been described as the states of anomalous rotational band of the non-axial rotor in the asymmetric rotor model (ARM). The ARM is capable of making predictions for any nucleus for which the first and second 2^+ energy states are known. The level structure predicted by ARM for $\gamma = 0^{\circ}$ or 60° is same as of the asymmetric rotor of Bohr-Mottelson model. Thus ARM appears to provide a somewhat more general description of the nucleus then by other collective models.

Turner et al.[3] applied the macroscopic point of view in high spin collective states assuming asymmetric nuclear shape. When prolate-oblate energy difference was small, the deviation from axial symmetry started. The phase transition of the rotational mode from axially symmetric to asymmetric shape was found to occur abruptly at the spin 8. Nuclear deformation at $\gamma = 30^{\circ}$ was such that the moment of inertia about one axis was maximized at the cost of other two. The lowest energy for high spin states were obtained by rotation of the nucleus about this axis when quantum fluctuation effects were unimportant.

The method of variation after angular momentum projection (VAP) was applied by Warke et al.[25] to the experimentally observed high spin states and their electromagnetic properties in even-even nuclei. The pairing plus quadrupole model of Baranger and Kumar[26] could explain the observed electromagnetic properties of the even-even nuclei, but they failed to describe the realistic even-even nuclei when the pairing forces were weak. Kota[27] has tried to explain the low lying spectra of some rare earth nuclei in the frame work of pseudo SU(3) model, but no attempt has been made to explain the high spin states. Sahu et al.[28] have considered a static tri-axial shape of the nucleus but the calculations have been performed in the frame work of Hartree BCS theory employing the pairing plus quadrupole interaction.

Tanabe et al.[8] have proposed a general formalism for describing microscopically the asymmetric and symmetric rotors in both even and odd mass nuclei. The particle number projected HFB theory has been applied by Fassler et al.[29] to the asymmetric deformations, but it provides an incomplete picture to calculate three moments of inertia without recourse of all the components of angular momentum and the full D_2 symmetry scheme. Toyama[30] has considered a collective Hamiltonian to be composed of the vibrational, rotational and potential energy terms. The calculated energy levels are, with few exceptions, in good agreement with their respective experimental values. The relative B(E2) values are much improved in comparison with corresponding values calculated from the Clbsch Gordan Coefficients.

Tamura et al.[31,32] have developed a theory, which describes spherical and deformed nuclei and consequently the transitional nuclei on an equal footing. They have solved the equations for the coefficients in the expansion of the fermion-pair operators in terms of boson operator exactly to sixth order. Their results are subject to the objection that the discrepancies between calculated and observed energy values are in opposite directions for 3^+ and 4^+ states of ^{154}Sm and ^{150}Sm and for 2^+ and 3^+ states of ^{152}Sm belonging to gamma band. Tamura fails misearably, in both quality and quantity for the transition ratio $4^{+\prime} \rightarrow 2^+/4^+$.

The validity of V.A.P. approach[25] has not yet been confirmed[33] as the transition probabilities depend primarily on deformation of the nucleus, while the moment of inertia is sensitive to both the deformation and the pairing interaction.

F.T. Baker[34] has extended the DF model by adding hexadecapole shape components and has applied it to some transitional nuclei. The electric quadrupole properties, previously thought to be anomalous have been correctly predicted. The results have been improved by including β_4 as an additional degree of freedom than corresponding inclusion of only β_2 and γ as parameters.

The ARM of Davydov-Filippov has been extended to odd mass nuclei[35] by coupling a single nucleon to an inert core of well stablized asymmetric equilibrium shape. Calculations showed that it was very difficult to distinguish between a symmetric and an asymmetric rotor model for small γ. Meyertervehn[4,5] used the Davydov approximation which fixed the collective wave function at its average values and assumed a rigid triaxial shape. The assumption of a fixed shape affects the odd A solution at two points (i) at the energies and wave functions of the core (ii) at the coupling of odd nucleons to the core. From the first point the spectrum of rigid triaxial rotor approximately reproduces the lowest excited states of even transitional nucleu i.e. in $135 < A < 190$ mass region. In particular it accounted for the low lying second 2^+ state which characterises the triaxial shapes. On the other hand there is systematic deviations which reflect the softness of these nuclei. The second point concerning the particle core coupling is more important with respect to odd A spectrum, because it determines the level order and at this point the approximation of a fixed triaxial shape is well supported by the comparison with experimental data. The odd A spectrum changes drastically in going from prolate ($0^o < \gamma < 30^o$) to oblate ($30^o < \gamma < 60^o$) shapes. In fact rather complex families of unique parity states can be reproduced with fixed values of β and γ, derived from the neighbouring even nuclei. Toki and Faessler[35)] included the softness of the core due to centrifugal stretching or the coriolis antipairing effects by generalizing the variable moment of inertia model. The asymmetric rotor description turns out to be more simpler for large deformations i.e. for strong rotation vibration coupling.

During the last decade many workers[36-40)] have calculated γ from the energy ratio $E4^+/E2^+$ and evaluated B(E2) values for deformed even-even nuclei in rare earth and actinide regions. While calculating the B(E2) ratios for the cascades in the cross over transition from the 2^+ vibrational states Puri et al.[39] have concluded that the DR model is better than DF model. This is against the fact since the inclusion of the coupling of rotation with beta vibrations worsens the agreement with experimental values as shown by Gupta et al.[41] Therefore, if γ were taken from $E4^+/E2^+$, then the energies of other rotational band are predicted to an accuracy well within 1% but energies of $2^{+'}$, 3^+, $4^{+'}$ levels can not be predicted with any precision. Hence the use of such values as done by some workers[37,40] has no justification. The method of deducing γ from $E2^+/E4^+$ as done by Meyertervehn[5] has already been commented as unreliable by Baker et al.[42]

In the present work we have introduced a new method for the calculation of the non-axiality parameter γ using $E2^+$ values and model dependent intrinsic quadrupole moment Q_o. This method rectifies the usual DF values of γ by enhancing it a few degree at about 15^o and reducing it a few degrees at $20^o < \gamma < 30^o$. A linear relationship between intrinsic quadrupole moment Q_o and non-axiality parameter γ observed by us has been used to predict the energies of gamma vibrational band with spins 2^+, 3^+, 4^+ and the meanlives of low lying energy levels.[43,44]

The view point of Zawischa et al.[13] regarding the nature of low-lying levels as non-collective has been contradicted in both gamma and beta vibrational bands by Gupta et al.[12) They have established that ARM estimates are much closer to their experimental values. In view of such major success of DF model in describing anomalous rotational band, Gupta et al.[12) have worked out a semi empirical relation depending on γ alone to predict B(E2) values for entire gamma ray cascade in the region $150 < A < 190$ and $A > 220$, within the frame work of ARM. The accuracy of data needed to test the model are taken to lie within a factor of two, as suggested by Kumar[47) (i.e. $0.5 <$ enhancement/hinderance factor $F < 2$). The empirical relation developed by Gupta et al.[12) has been modified by employing ARM dependent intrinsic quadrupole moment Q_0 instead of experimentally measured Q_0. This modified empirical relation correctly reflects the asymmetric character. In addition, a number of medium mass nuclei in the range $28 \leqslant Z \leqslant 50$ and $50 \leqslant N \leqslant 82$, the new deformation region, possessing $15^\circ < \gamma < 30^\circ$ have been included which truely describes the asymmetric properties.

The new approach of determining γ has been applied to study electromagnetic properties of Samarium isotopes and the résults obtained have been compared with calculations of Weeks and Kumar.[26,31) The study of nuclear shape with the increase of spin in the ARM frame work has been done and it is inferred that the abrupt change in the rigid shape of nucleus takes place at I = 6. The most important part of the work is the prediction of B(E2) values, meanlives of rotational and gamma band and the beta band head energy $EO^{+'}$ which, if known, are compared with experimental data and an excellent agreement has been observed. The experimental data for input parameters are taken from the Table of Isotopes[48)] and Sakai-Rester Table[49)].

The importance of the work lies in the prediction of B(E2) values and meanlives of rotational as well as gamma bands and the beta band head energies ($EO^{+'}$) in respect of deformed even-even nuclei belonging to medium and heavy mass regions, since the measurement of B(E2) values and meanlives of excited states are one of the most active areas of nuclear structure physics. During the last decade it has also been a major research point in devising new experimental techniques which can provide data giving unique and vital information and playing a major role in our understanding of nuclear structure. A knowledge of approximate values of meanlives predicted by us is also of great importance to the experimentalists.

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CHAPTER - II

REVIEW OF THEORETICAL APPROACHES

2.1 Introduction:

Nuclear structure is generally explained on the basis of two nuclear models, the shell model and the collective model. In the shell model one performs detailed calculations with specific potentials and residual interactions to explain not only general features displayed by nuclei over fairly wide regions but also to explain and predict properties of individual nuclei. In the collective model one considers quantized fluid with certain properties and subjects it to certain boundary conditions in order to predict gross characteristics which depend upon most of the nucleons in a system. In order to test the validity of such models, generally a survey of selected nuclear properties like binding energies, excitation energies, nuclear moments, transition probabilities etc. is done. It is well known that different nuclear models are applicable to different regions of periodic table with varying degrees of success in explaining different nuclear properties.

2.2 A Brief Description of Different Nuclear Models:

2.2(a) Nuclear Shell Model:

The shell model of nucleus was proposed by Meyer and Jensen[1)] to explain large amount of low energy experimental data. In this model it is assumed that the nucleons in the nucleus move independently in a common static potential, determined by the average motion of all other nucleons. The nucleons are supposed to be arranged in regular shells obeying Pauli's exclusion principle. Most of the nucleons are paired so that a pair of nucleons contributes zero spin and zero magnetic moment. The properties of odd A nuclei by the unpaired proton and neutron. Two types of potentials i.e. infinite square well and harmonic oscillator potential are considered. The addition of the spin-orbit term eliminates some difficulties experienced with the above two potentials. Mayer and Jensen suggested that a non-central component should be included in the force acting on a nucleon in a nucleus to yield the observed magic numbers. It is corresponding to the interaction between the orbital angular momentum and the intrinsic angular momentum. The nuclear potential experienced by a single nucleon within the nucleus due to remaining nucleons is given by

$$V_{(r)} = V_{o(r)} + V_{1(r)}(\vec{\sigma}.\vec{\ell}) + \sum_{i} V_{ik}(r_{ik}) \qquad [2.1]$$

The first term is due to usual central potential, the second term is due to spin orbit coupling and the third term is due to residual interaction of all other nucleons.

In many particle shell model all A nucleons in the nucleus are taken into account. Each nucleon moves independently of all other nucleons in a common potential field, hence is considered as independent particle.

The shell model has been very successful in predicting the ground state spin of a large number of nuclei. Neutron and proton levels fill independently according to Pauli exclusion principle. Even-even nucleus has total ground state angular momentum equal to zero. For a nucleus with odd Z or odd N, the nucleons pair off as far as possible, so that the resulting orbital angular momentum and spin directions are just that of the single odd particle. There are some exceptions to this also. In odd-odd nucleus, the total angular momentum is the vector sum of the odd neutron and odd proton values. The parity is the product of proton and neutron parity. In an odd nucleus, the magnetic moment of the nucleus is produced by the odd nucleon only and the electric quadrupole moment due to odd Z nucleon. Independent particle model can explain the experimental facts, like magnetic moments of excited nucleus, ground state spins and low-lying level spectnum.

The deviation of magnetic moments from the Schmidt curves make this model less applicable. The measured quadrupole moments are several times larger than can be attributed to the odd nucleon even in nuclei with just one nucleon more or less than a closed shell, where the single particle model should be at its best. In addition to this the model can not explain the ground state spins in the mass regions $150 < A < 190$ and $A > 220$ and transition probabilities in the case of even-even nuclei.

2.2(b) Bohr-Mottelson Model:

The most striking evidence of the collective phenomenon in nuclei was provided by the existance of rotational bands in the nuclear spectra observed in the Coulomb excitation studies. The Coulomb excitation studies also revealed the systematic occurance of strong E2 transition in nuclei outside the rotational regions. The low energy spectra of even-even nuclei outside the rotational regions could be interpreted, atleast qualitatively, in terms of quadrupole vibrations about a spherical equilibrium. Large values of quadrupole moments can be obtained if the nuclei are assumed to be deformed so that they have permanent non-spherical shapes (spheroidal shapes). Rainwater[2)] noted that the many number of protons in nucleus can give large value of electric quadrupole moments as a result of the polarizing action of one or more loosely bound nucleons on the remaining nucleus, which is the basis of collective behaviour.

Bohr-Mottelson[3,4)] have developed a single model, called the unified model. It is an extension of shell model. In this model the shell model potential is assumed non-spherical. The energies of the single particle in this non-spherical potential are calculated, and the distortion which gives the minimum energy is taken as the actual distortion. The long range correlations are replaced by the assumption of a permanently distorted potential. The model necesserily represents the collective effects of the nucleons in the nucleus.

(i) Nulcear Rotational Motion

The collective motion of the nucleus is more evident. when one considers the excitations of the even-even nuclei. The collective rotational motion of the nucleus which has axial symmetry is similar to the rotation of a symmetric top. There are two sets of orthogonal systems of axes (i) body fixed reference frame with 1,2,3 axes and (ii) laboratory fixed reference frame with x,y,z axes. In first system the body fixed reference frame is attached to the rotating body. The 3-axis is used as the axis of symmetry. If $\mathcal{I}_3$ and $\mathcal{I}$ are the moments of inertia for rotations about symmetry 3-axis and about an axis perpendicular to it and I is the total angular momentum operator with components I_1, I_2 and I_3 along the body fixed axes, the Hamiltonian is expressed[5)]

$$H = \sum_{i=1}^{3} \frac{\hbar^2}{2\mathcal{I}_i} I_i^2 = \frac{\hbar^2}{2\mathcal{I}} [I^2 - I_3^2] + \frac{\hbar^2}{2\mathcal{I}_3} I_3^2 \qquad [2.2]$$

where, for the symmetric top, $\mathcal{I}_1 = \mathcal{I}_2 = \mathcal{I}$

The spherical harmonic $Y_{\ell m}(\Omega)$ are expressed as

$$Y_{\ell m}(\Omega) = \sum_{m'=-\ell}^{\ell} Y_{\ell m'}(\Omega') D^{\ell}_{mm'}(\theta,\phi,\psi) \qquad [2.3]$$

where Ω and Ω' are the initial and final polar angles (θ,ϕ) and (θ',ϕ') and m and m' can take independently any of the values $-\ell$, $-\ell+1, \ldots\ldots \ell-1, \ell$.

The nuclear wave functions are the D-functions which are the transformation functions for the spherical harmonics under finite rotations. The spherical harmonics are expressed in equation (2.3) under a rotation through Euler angles θ, ϕ and ψ (counter clockwise rotation $0 \leqslant \theta < 2\pi$ about the z-axis followed by a rotation $0 \leqslant \phi \leqslant \pi$ about the new y-axis and a rotation $0 \leqslant \psi \leqslant 2\pi$ about the new 3-axis).

Let M be the component of the angular momentum I along the z-axis and K be the component of I along the symmetry axis-3 as shown in fig. 2.1, Then we may obtain the following relations satisfied by the angular momentum operator

$$I^2 D^I{}_{MK} = I(I+1) D^I{}_{MK} \qquad [2.4]$$

$$I_2 D^I{}_{MK} = M D^I{}_{MK} \qquad [2.5]$$

$$I_3 D^I{}_{MK} = K D^I{}_{MK} \qquad [2.6]$$

A normalized wave function corresponding to the Hamiltonian of equation (2.2) is expressed as

$$\psi^I{}_{MK} \equiv | IMK > = [\frac{2I+1}{8\pi^2}]^{1/2} D^I{}_{MK} (\theta, \phi, \psi) \qquad [2.7]$$

and the energy eigen values are

$$E_{IK} = \frac{\hbar^2}{2\mathcal{I}}[I (I+1) - K^2] + \frac{\hbar^2}{2\mathcal{I}_3} K^2 \qquad [2.8]$$

(ii) Energy Eigen Values:

For axially symmetric even-even nuclei K = 0, because even-even spherical nuclei do not show rotational spectra and

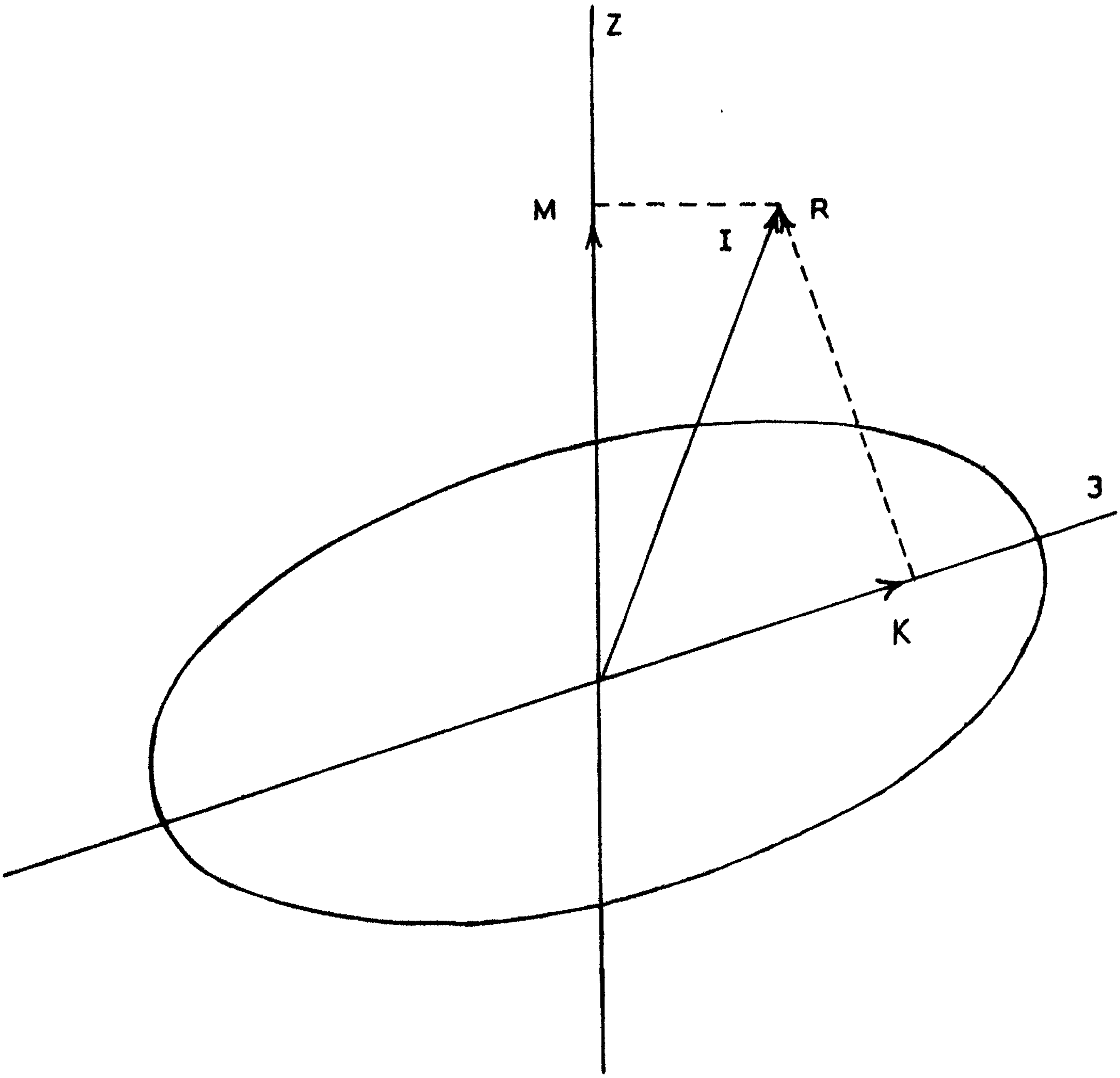

Fig.2.1 Schematic diagram for angular momenta in deformed nuclei

hence do not rotate about the axis of symmetry, thus the angular momentum about a symmetry axis becomes zero. The wave function and energy spectrum for even-even nuclei are given by

$$\psi^{I}_{M0} = |IM0> = i^{-M}\left[\frac{2I+1}{8\pi^2}\right]^{1/2} D^{I}_{M0}(\theta,\phi) \qquad [2.9]$$

$$= Y_{IM}(\theta,\phi)$$

and

$$E_I = \frac{\hbar^2}{2\mathcal{I}}\left[\, I(I+1)\,\right] \qquad [2.10]$$

where the factor i^{-M} has been inserted in equation (2.9) so that ψ^{I}_{M0} corresponds to Y_{IM}.

Since a spheroid is invariant under a rotation of π about any axis passing through the centre, the wave function expressed in equation (2.7) should be invariant with respect to rotation by π about, say the 1-axis, that is, under a transformation of the Euler angles

$$\theta \to \pi + \theta, \quad \phi \to \pi - \phi \qquad \psi \to \psi$$

we represent this rotation by an operator

$$R_1(\theta \to \pi + \theta,\ \phi \to \pi - \phi,\ \psi \to \psi)$$

Invariance under R_1 requires that the normalized wave function be

$$\psi^{I}_{MK} \equiv |IMK> = \left[\frac{2I+1}{8\pi^2}\right]^{1/2} \frac{1}{\sqrt{2}}\,(1+R_1)D^{I}_{MK}(\theta,\phi,\psi)$$

$$= \left[\frac{2I+1}{16\pi^2}\right]^{1/2}\left[D^{I}_{MK} + (-1)^{I+K} D^{I}_{M,-K}\right] \qquad [2.11]$$

Since

$$R_1 D^I_{MK}(\theta,\phi,\psi) = e^{i\pi(I+K)} D^I_{M,-K}(\theta,\phi,\psi) \qquad [2.12]$$

$$R_1^2 = 1 \qquad R_1 \psi^I_{MK} = \psi^I_{MK}$$

from equation (2.11), it follows that

$$\text{if } K = 0, \text{ for } \psi^I_{MK} \neq 0, \text{ we require } I = 0,2,4\ldots.. \qquad [2.13]$$

In other words for K = 0 only even angular momentum states of I are allowed.

If the nucleus has additional (internal) degrees of freedom associated with rotational motion, denoted by χ_i, then the correct wave function replacing the equation (2.11) is the product wave function

$$\psi^I_{MK} \equiv |IMK> = [\frac{2I+1}{16\pi^2}]^{1/2}[D^I_{MK}\chi_i + (-1)^{I+K} D^I_{M,-K} R_1 \chi_i] \qquad [2.14]$$

where we have assumed that the total wave function is the product of two wave functions.

A large amount of data is available for even-even nuclei with spin sequence 0^+ for the ground state, 2^+ for the first excited state, 2^+ or 4^+ for the second excited state for axially symmetric nuclei. The rotational energy spectrum given by equation (2.8) with K = 0 for even-even nuclei is explained well by the equation (2.10). However, there is a systematic deviation from the rotational spectrum, which can be explained by adding a correction term of the form $I^2(I+1)^2$. Thus in the next approximation, the energy is expressed as

$$E_I = \frac{\hbar^2}{2\mathcal{J}} I(I+1) - BI^2(I+1)^2 \qquad [2.15]$$

the value of B can be evaluated from the energy of the 4^+ state, which is approximately given by $\frac{\hbar^2}{2\mathcal{J}} \times 10^{-3}$ for even-even nuclei with $150 < A < 190$. The deviation from the rotational spectrum may be thought of as due to weak coupling between a rotational mode and either vibrational or particle mode.

(iii) Reduced Electric Transition Probabilities:

The reduced electric quadrupole transition probability $B(E2: I \rightarrow I')$ between two members of the same rotational band with quantum number K is expressed as

$$B(E2: IK \rightarrow I'K) = \frac{5}{16\pi} e^2 Q_0^{\,2} \, |\langle I2K0 | I'K \rangle|^2 \qquad [2.16]$$

where we have used

$$\sum_{M_1, M_2, M} |\langle I_1 I_2 M_1 M_2 | IM \rangle|^2 = (2I+1) \qquad [2.17]$$

For coulomb excitation, the B(E2), reduced transition probability in the case of a symmetric rotor (even-even nuclei) is expressed as

$$B(E2: I \rightarrow I+2) = \frac{5}{16\pi} e^2 Q_0^{\,2} \, |\langle I200 | I+2,0 \rangle|^2$$

$$= \frac{15}{32\pi} e^2 Q_0^2 \frac{(I+1)(I+2)}{(2I+1)(2I+3)} \qquad [2.18]$$

The non-spherical nuclei have rotational levels which are due to very fast electric quadrupole transition probability $B(E2: I \rightarrow I')$.

According to equation (2.16), B(E2: $I \rightarrow I'$) increases as the value of intrinsic quadrupole moment Q_o increases.

If the transition takes place between the ground state (I = 0) and the first excited state (I = 2) of even-even nuclei, then

$$B(E2) \;=\; \frac{5}{16\pi}\, e^2 {Q_o}^2 \qquad [2.19]$$

(iv) Nuclear Deformation:

If the nucleus is considered as a uniformly charged spheroid, then taking the radial coordinate of the surface of the nucleus as

$$R \simeq R_o\,[\;1 + \beta\; Y_{20}\,(\theta,\phi\,)] \qquad [2.20]$$

the intrinsic quadrupole moment Q_o is given by

$$Q_o = e \int d^3\, r\rho(r)\; < IM \mid\; (3z^2 - r^2) \mid\; IM >_{M=I} \qquad [2.21]$$

$$= \frac{3}{\sqrt{5\pi}}\, Ze\; R_o^2\; \beta[\;1 + 0.36\beta + \;........] \qquad [2.22]$$

The deformation parameter β can be determined with the observed value of Q_o. The deformation parameter β can be related to ΔR, the difference between the major and minor semi-axes as

$$\beta \;=\; \frac{4}{3}\sqrt{\frac{\pi}{5}}\;\frac{\Delta R}{R_o} = 1.06\;\frac{\Delta R}{R_o} \qquad [2.23]$$

(v) Collective Vibrational Excitations:

The nucleus can perform small oscillations about the equilibrium shape, if it is considered to be a dynamic system.

The oscillations can be analysed in terms of normal modes and can be treated as independent for small oscillation amplitudes. Since the energies of vibrational excitations are of the order of several hundred keV to few MeV, the coupling between the vibrational and intrinsic motions is no longer weak, as is the case with rotational motion. Thus the agreement of the experimental data with the theoretical predictions in vibrational spectra would not be as good as in rotational spectra. The vibrational states are known to occur not only among spherical even-even nuclei but also among deformed nuclei.

In even-even nuclei, the β- and γ-vibrations are well known. The beta vibrations have spin 0 and can be pictured as the vibrations conserving rotational symmetry. The gamma vibration is a quadrupole oscillations in the spin 2. The parameters β and γ are known as the deformation and non-axiality parameters. These vibrational levels can be regarded as originating from the two phonon states in the spherical nuclei, but the relative energy shifts are large. The nucleus is considered as an incompressible liquid drop with a sharp surface, and the nuclear wave function is described interms of the radius vector specifying the nuclear surface. If R_o be the radius of spherical nucleus the equation for the surface can be written[2,3)

$$R\ (\theta,\phi\ ) = R_o\ [\ 1 + \sum_{\lambda\mu} \alpha_{\lambda\mu}\ Y^{*}_{\lambda\mu}(\theta,\phi\ )] \qquad [2.24]$$

where $Y_{\lambda\mu}$ are the spherical harmonics and $\alpha_{\lambda\mu}$, the deformation parameter which determines the nuclear shape. The subscript μ takes the values $-\lambda$ to $+\lambda$, so that there are $2\lambda+1$ modes of deformation of order λ. The lowest mode of surface deformation corresponds to quadrupole mode ($\lambda=2$). The mode with $\mu=0$ (for all λ values) has symmetry with respect to arbitrary rotation about the z-axis and therefore represents an axially symmetric nuclear shape.

(vi) Quadrupole Deformation:

Considering the ellipsoidal shape and restricting to the quadrupole deformation $\lambda=2$ in equation (2.24) the surface of the ellipsoid is described by $R(\theta',\phi')$ and the deformation $\delta R(\theta',\phi')$ from the radius R_o of the sphere (of the same volume) is given by

$$\delta R(\theta',\phi') = R(\theta',\phi') - R_o$$

$$= R_o \sum_{\mu=-2}^{2} \alpha_{2\mu}^{*} Y_{2\mu} \qquad [2.25]$$

where the deformation parameter $\alpha_{2\mu}$ and the polar angles θ',ϕ' are with respect to laboratory system. Since δR is real, we get

$$\alpha_{2-\mu} = (-1)^{\mu} (\alpha_{2\mu})^{*} \qquad [2.26]$$

From equation (2.24) it is clear that there are only five independent deformation constants $\alpha_{2\mu}$ ($\mu=-2$ to $+2$), two specifying the shape and three describing the orientation.

In the body fixed reference frame in which the coordinate axes coincide with the principal axes, the deformation parameter $\alpha_{2\mu}$ is denoted by $a_{2\mu}$. Relationship between deformation in two coordinate system is

$$\sum_{\mu} a_{2\mu} Y_{2\mu}(\theta,\phi) = \sum_{\nu} a_{2\nu}^{*} Y_{2\nu}(\theta,\phi)$$

$$= \sum_{\nu} a_{2\mu}^{*} \sum_{\mu} D_{\mu\nu}^{2} (\theta,\phi,\psi) Y_{2\mu}(\theta',\phi') \qquad [2.27]$$

so that $$a_{2\mu} = \sum_{\nu} a_{2\nu} D_{\mu\nu}^{*2} (\theta,\phi,\psi) \qquad [2.28]$$

Since the product of inertia is zero interms of the principal axes, we define the following

$$a_{20} = \beta \operatorname{Cos} \gamma \qquad [2.29]$$

$$a_{21} = a_{2,-1} = 0 \qquad [2.30]$$

$$a_{22} = a_{2,-2} = 1/\sqrt{2}\, \beta \operatorname{Sin} \gamma \qquad [2.31]$$

The deformation δR_j along the principal axes j = 1,2,3 (i.e. the nuclear body fixed axes) are obtained from

$$\delta R(\theta,\phi) = R_o \sum_{\mu=-2}^{2} a_{2\mu}^{*} Y_{2\mu}(\theta,\phi) \qquad [2.32]$$

and are given by

$$\delta R_1 \left(\frac{\pi}{2},0\right) = \left(\frac{5}{4\pi}\right)^{1/2} \beta R_o \operatorname{Cos}\left(\gamma - \frac{2\pi}{3}\right) \qquad [2.33]$$

$$\delta R_2 \left(\frac{\pi}{2},\frac{\pi}{2}\right) = \left(\frac{5}{4\pi}\right)^{1/2} \beta R_o \operatorname{Cos}\left(\gamma - \frac{4\pi}{3}\right) \qquad [2.34]$$

$$\delta R_3 (0,\phi) = \left(\frac{5}{4\pi}\right)^{1/2} \beta R_o \operatorname{Cos} \gamma \qquad [2.35]$$

the parameter β is a measure of deviation from sphericity and γ determines the shape of the nucleus.

2.2(c) Davydov-Filippov Model

Davydov and Filippov[6-8] have suggested a model for the deformed nuclei. In this model they assume that the rotation of the nucleus takes place without a change of its intrinsic state. The equilibrium shape of the nucleus is determined by the parameters β and γ which are related to the body fixed coordinates a_{μ} defined in equations (2.29) (2.30) and (2.31). The three axes of the ellipsoid used to described the shape of the nucleus are given by

$$R_j = R_o \left\{ 1 + \left(\frac{5}{4\pi} \right)^{1/2} \beta \operatorname{Cos} \left(\gamma - \frac{2j\pi}{3} \right) \right\} \qquad [2.36]$$

$$j = 1,2 \text{ and } 3$$

For the fixed value of β, varying γ from 0 to $\pi/3$, the shape of the nucleus changes from a prolate to an oblate ellipsoid. $\gamma = 30^o$ corresponds a shape between prolate and oblate ellipsoid of revolution. Such an axially asymmetric model has been found quite suitable in explaining the rotational levels of the deformed even-even nuclei, the large electric quadrupole moments and the transition probabilities.

The energies of the various excited states in non-axial nuclei can be computed by an operator of the form

$$H = \sum_{\lambda=1}^{3} \frac{A \, I_{\lambda}^2}{2 \operatorname{Sin}^2 \left(\gamma - \frac{2\pi\lambda}{3} \right)} \qquad [2.37]$$

where $A = \hbar^2 / 4B\beta^2$ is a quantity having the dimensions of energy, γ determines the deviation of the shape of the nucleus from axial symmetry and varies from 0 to $\pi/3$ and I_λ are the operators of the projections of the nuclear angular momenta on the axes of a coordinate system connected with the nucleus.

It is obvious from equation (2.37) that, for values of other than 0 and $\pi/3$, the nucleus should be regarded as an asymmetric top. Each value of angular momentum I is an asymmetric top corresponding to (2I+1) energy levels. Because of the symmetry conditions imposed on the wave function, the only allowed values of I for even nuclei are those which correspond to a completely symmetric representation of the group D_2. There are no rotational states of the required symmetry for I = 1. There are two states for I = 2, one for I = 3, three for I = 4 and two for I = 5 etc. The energies in units of $\hbar^2/4B\beta^2$ of the two levels of required symmetry for I = 2 are

$$E2^{+} = \frac{9[1 - \{1 - \frac{8}{9}\sin^2(3\gamma)\}^{1/2}]}{\sin^2(3\gamma)}, \qquad [2.38]$$

$$E2^{+'} = \frac{9[1 + \{1 - \frac{8}{9}\sin^2(3\gamma)\}^{1/2}]}{\sin^2(3\gamma)} \qquad [2.39]$$

for I = 3, the energy level is

$$E3^{+} = \frac{18}{\sin^2(3\gamma)} \qquad [2.40]$$

From equations (2.38), (2.39) and (2.40) we get

$$E2^{+} + E2^{+'} = E3^{+} \qquad [2.41]$$

The computed energy levels for various values of γ are given in Fig. 2.2. For $\gamma = 0$, we obtain the energy levels corresponding to axially symmetric nuclei treated by Bohr and Mottelson[4).

The Davydov and Filippov model also predicts the transition probabilities between the rotational state of the second excited state of spin 2 and the first excited state of spin 2 or to the ground state of spin 0 of an even nucleus. The spectroscopic quadrupole moment Q(I) and the reduced transition probabilities $B(E2: I_i \rightarrow I_f)$ are function of Q_o defined by equation (2.22) and parameter γ e.g.

$$Q_{(2)} = - Q_{(2')} = - Q_o \frac{6 \text{ Cos } (3\gamma)}{7[\ 9-8 \text{ Sin}^2(3\gamma)]^{1/2}} \qquad [2.42]$$

Similarly the quadrupole transition probabilities are given by

$$B(E2:\ 2^+ \rightarrow 0^+) = \frac{e^2 Q_o^{\ 2}}{32\pi} [1 + \frac{3-2 \text{ Sin}^2(3\gamma)}{\{9-8 \text{ Sin}^2(3\gamma)\}^{1/2}}] \qquad [2.43]$$

$$B(E2:\ 2^{+'} \rightarrow 0^+) = \frac{e^2 Q_o^{\ 2}}{32\pi} [\ 1 - \frac{3-2 \text{ Sin}^2(3\gamma)}{\{9-8 \text{ Sin}^2(3\gamma)\}^{1/2}}] \qquad [2.44]$$

$$B(E2:\ 2^{+'} \rightarrow 2^+) = \frac{10}{7} \frac{e^2 Q_o^{\ 2}}{16\pi} [\ \frac{\text{Sin}^2(3\gamma)}{9 - 8 \text{ Sin}^2(3\gamma)}\ ] \qquad [2.45]$$

These equations give satisfactory agreement with the experimental data. Equation (2.43) and (2.44) allow us to express Q_o in terms of the easily observable transition probabilities $B(E2:\ I_i \rightarrow I_f)$:

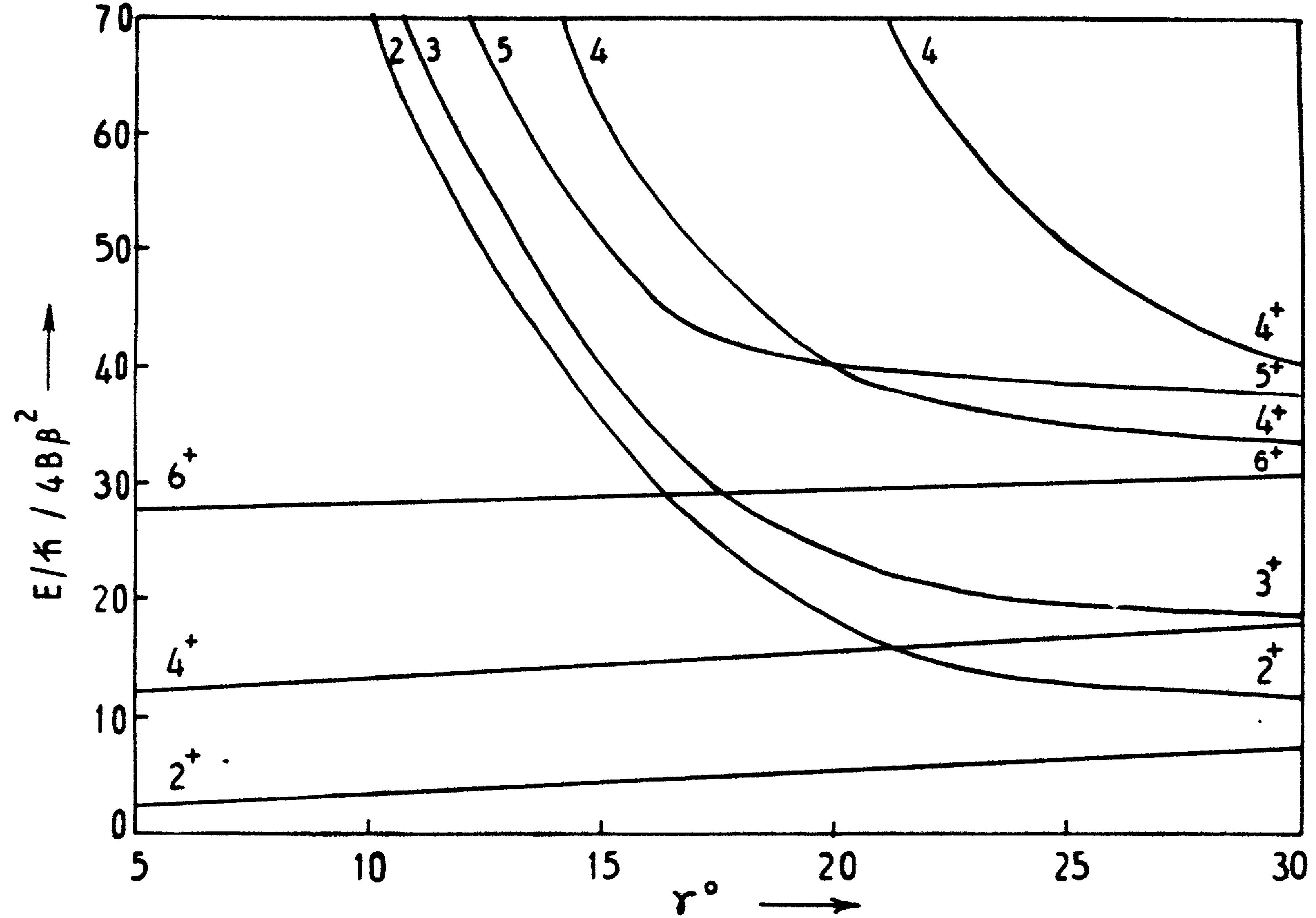

Fig. 2.2 The computed energy levels for various values of the non-axiality parameter γ.

$$e^2Q_o^2 = 16\pi\,[B(E2: 2^+ \to 0^+) + B(E2: 2^{+\prime} \to 0^+)] \qquad [2.46]$$

and provides us with a (model dependent) method of determining quadrupole moment of excited states.

The factor $(e^2Q_o^2/16\pi)$ is the reduced electric quadrupole transition probability between rotational levels of spin 2 and 0 for axially symmetric nuclei.

The DF model has predicted successfully many rotational levels including those of higher excitations. The expression[7] for reduced electric quadrupole transition probabilities between two states described by the functions ψ_{Imi} and $\psi_{I'm'f}$ as

$$B(E2: i \to f) = \frac{5}{16\pi(2I+1)} \sum_{\mu m' m} |(I'm'f|\hat{Q}_{2\mu}|\ Imi\ )|^2 \qquad [2.47]$$

where $$\hat{Q}_2 = eQ_o[\ D^2_{\mu o}\ \mathrm{Cos}\,\gamma + \frac{\mathrm{Sin}\,\gamma}{\sqrt{2}}(D^2_{\mu 2} + D^2_{\mu -2})] \qquad [2.48]$$

and $$Q_o = \frac{3Z\ R^2\beta}{\sqrt{5\pi}} \qquad [2.49]$$

is the intrinsic quadrupole moment of an axial nucleus with a deformation parameter β. The generalized spherical functions $(D^2_{\mu\nu})$ define the unitary transformation from a coordinate system fixed in space to a coordinate system fixed to the nucleus.

The wave functions of the rotational states of a non-axial nucleus can be written in the form

$$\psi_0 = \frac{1}{\sqrt{8\pi^2}}\Phi(\beta,\gamma\ ) \qquad [2.50]$$

$$\psi_{21m} = \left(\frac{5}{8\pi^2}\right)^{1/2} \Phi(\beta,\gamma)\left[a_1 D^2_{mo} + b_1 \frac{D^2_{m2} + D^2_{m-2}}{\sqrt{2}}\right] \qquad [2.51]$$

$$\psi_{22m} = \left(\frac{5}{8\pi^2}\right)^{1/2} \Phi(\beta,\gamma)\left[a_2 D^2_{Mo} + b_2 \frac{D^2_{M2} + D^2_{M-2}}{\sqrt{2}}\right] \qquad [2.52]$$

where $\Phi(\beta,\gamma)$ is a function corresponding to the internal state of a nucleus which is assumed to be the same in all three rotational states. Also

$$a_1 N_1 = -\{\mathrm{Sin}\,\gamma\,\mathrm{Sin}(3\gamma) + 3\mathrm{Cos}\,\gamma\,\mathrm{Cos}(3\gamma) + [9-8\mathrm{Sin}^2(3\gamma)]^{1/2}\}$$

$$b_1 N_1 = 3\mathrm{Sin}\,\gamma\,\mathrm{Cos}(3\gamma) - \mathrm{Cos}\,\gamma\,\mathrm{Sin}(3\gamma)$$

$$N_1^{\,2} = 2\{9-8\mathrm{Sin}^2(3\gamma)\}^{1/2}[\{9-8\mathrm{Sin}^2(3\gamma)\}^{1/2} + \mathrm{Sin}\,\gamma\,\mathrm{Sin}(3\gamma) + 3\,\mathrm{Cos}\,\gamma\,\mathrm{Cos}(3\gamma)] \qquad [2.53]$$

$$a_2 N_2 = \{9-8\mathrm{Sin}^2(3\gamma)\}^{1/2} - \mathrm{Sin}\,\gamma\,\mathrm{Sin}(3\gamma) - 3\,\mathrm{Cos}\,\gamma\,\mathrm{Cos}(3\gamma)$$

$$b_2 N_2 = 3\,\mathrm{Sin}\,\gamma\,\mathrm{Cos}(3\gamma) - \mathrm{Cos}\,\gamma\,\mathrm{Sin}(3\gamma)$$

$$N_2^{\,2} = 2\{9-8\mathrm{Sin}^2(3\gamma)\}^{1/2}[\{9-8\,\mathrm{Sin}^2(3\gamma)\}^{1/2} - \mathrm{Sin}\,\gamma\,\mathrm{Sin}(3\gamma) - 3\,\mathrm{Cos}\,\gamma\,\mathrm{Cos}(3\gamma)] \qquad [2.54]$$

The wave functions of states with spin 4 and 6 are

$$\psi_{4mi} = \left(\frac{9}{8\pi^2}\right)^{1/2}\left\{A_{oi} D^4_{mo} + A_{2i} \frac{D^4_{m2} + D^4_{m-2}}{\sqrt{2}} + A_{4i} \frac{D^4_{m4} + D^4_{m-4}}{\sqrt{2}}\right\} \qquad [2.55]$$

$$\psi_{6mi} = \left(\frac{13}{8\pi^2}\right)^{1/2}\left\{B_{oi} D^6_{mo} + B_{2i} \frac{D^6_{m2} + D^6_{m-2}}{\sqrt{2}} + B_{4i} \frac{D^6_{m4} + D^6_{m-4}}{\sqrt{2}} + B_{6i} \frac{D^6_{m6} + D^6_{m-6}}{\sqrt{2}}\right\} \qquad [2.56]$$

By using equation (2.47) and the wave functions of states with spin 4 and 2, one can compute the reduced transition probabilities between these states. Thus the reduced probability (in $e^2 Q_o^2/16\pi$ units) for the quadrupole transition between spin 4 and spin 2 levels can be expressed

$$B(E2:\ 4i \rightarrow 2f) = \frac{5}{126}\{\mathrm{Cos}\,\gamma(6A_{oi}af + \sqrt{15}\,A_{2i}bf) + \mathrm{Sin}\,\gamma(\sqrt{15}\,A_{2i}af + A_{oi}bf + \sqrt{35}\,A_{4i}bf)\}^2 \quad [2.57]$$

where af and bf are coefficients[6] specifying the wave functions of states possessing a spin 2.

Finally the expression for the reduced quadrupole electric transitions between levels with spins 6 and 4

$$B(E2:\ 6i \rightarrow 4f) = \frac{5}{143}\{\mathrm{Cos}\,\gamma[3\sqrt{5}\,B_{oi}A_{of} + 4\sqrt{2.1}\,B_{2i}A_{2f} + 3B_{4i}A_{4f}] + \mathrm{Sin}\gamma[\sqrt{3}\,B_{oi}A_{2f} + \frac{B_{2i}A_{4f}}{\sqrt{10}} + \sqrt{14}B_{2i}A_{of} + \sqrt{21}\,B_{4i}A_{2f} + \sqrt{49.5}\,B_{6i}A_{4f}]\}^2 \quad [2.58]$$

2.2(d) <u>Davydov-Rostovsky Model</u>

In Davydov-Rostovsky model[10], the operator for collective excitations corresponding to the quadrupole vibrations of nuclear surface is expressed as

$$H = \frac{\hbar^2}{2B}\left[\,T_\beta + \frac{1}{\beta^2}(T_\gamma + T_{rot})\right] + V(\beta,\gamma) \quad [2.59]$$

where $V(\beta,\gamma)$ is the potential energy of β and γ-vibrations.

and $T_\beta = -\,\beta^{-4}\,\frac{\partial}{\partial\beta}(\beta^4\,\frac{\partial}{\partial\beta})$

$$T_\gamma = - \frac{1}{\mathrm{Sin}(3\gamma)} \frac{\partial}{\partial\gamma}[\ \mathrm{Sin}(3\gamma)\frac{\partial}{\partial\gamma}\ ]$$

$$T_{rot} = \frac{1}{4}\sum_{\ell=1}^{3} A_\ell\ (\gamma)\ I_\ell^2 \qquad [2.60]$$

and I_ℓ are the projections on the main nuclear axes of I, the total angular momentum operator. The dependence of the functions $A_\ell(\gamma)$ on the dynamic variable γ is given by the expressions

$$A_\ell = \{\ \mathrm{Sin}\ (\gamma - \frac{2\pi\ell}{3})\}^{-2}:\ \ell = 1,2,3 \qquad [2.61]$$

For nuclei in which the vibrations occur about an equilibrium spherical shape, this dependence is obtained[3] from the canonical transformation to the variable θ_1, θ_2, θ_3, β and γ of the operator,

$$H = \frac{1}{2}\sum_\mu \{\ C|\alpha_\mu|^2 + B|\dot{\alpha}_\mu|^2\ \} \qquad [2.62]$$

$$\mu = 0,\ \pm 1,\ \pm 2$$

of the collective quadrupole vibrations of the nuclear surface. The eigen functions of the operator (2.47) are functions of internal dynamic variables β and γ and three Euler angles $[\theta_1, \theta_2, \theta_3] \equiv \Theta$, and they are determined in the space element

$$d\tau = \beta^4\ |\ \mathrm{Sin}(3\gamma)\ |\ \mathrm{Sin}\theta_2\ d\beta\ d\gamma\ d\theta_1,\ d\theta_2\ d\theta_3 \qquad [2.63]$$

The collective excitations described by the operator (2.47) have not been investigated in a complete form. Certain simplification of operator (2.47) were solved in all previous investigations[3,4,9] Thus the nuclear excitations are given by the operator

$$H_\gamma = -\frac{\hbar^2}{2B_\gamma}\left[\frac{1}{\gamma}\frac{\partial}{\partial\gamma}\left(\gamma\frac{\partial}{\partial\gamma}\right) - \frac{I_3^2}{\gamma^2}\right] + \frac{1}{2}C_\gamma'\gamma^2 \qquad [2.64]$$

which is obtained from equation (2.47) under conservation of γ-dependent terms with the following additional simplifications.

$$B\beta^2 \to B\beta_o^2 = B_\gamma$$

$$V(\beta,\gamma) \to \frac{1}{2}C(\beta-\beta_o)^2 + \frac{1}{2}C_\gamma'\gamma^2$$

$$\frac{\hbar^2}{2B\beta^2}T_{rot} \to \frac{\hbar^2}{6B\beta^2}(I^2 - I_3^2) + \frac{\hbar^2}{2B_\gamma}\frac{I_3^2}{\gamma^2} \qquad [2.65]$$

According to equation (2.53), the second term in square brackets of operator (2.52) is a part of the rotational operator. If K = 0,2,4..... denotes the eigen values of the operator I_3, in the states with $K \neq 0$, the eigen values of operator (2.52) characterize complex excitations in which γ-vibrations and nuclear rotation about the axis 3 are inseparable. In particular the wave functions of these states depend on γ and the angle θ_3. Thus the excited states given by equation (2.52) can be called γ-vibrations only in the states with K=0. On the other hand, in DF adiabatic theory[6)], the excited states were described by the operator

$$H' = \frac{\hbar^2}{2B\beta^2}T_{rot} = \frac{\hbar^2}{8B\beta^2}\sum_{\ell=1}^{3} I_\ell^2\left\{\sin\left(\gamma - \frac{2\pi\ell}{3}\right)\right\}^{-2} \qquad [2.66]$$

in which the dynamic variables β and γ were replaced by their certain effective values. The corresponding excited states were called rotational though actually they also combine rotations with nuclear surface vibrations. This adiabatic approximation is satisfactory if one considers the excited state of ground band only and the first term of anomalous rotational band and the transition probabilities between them.

In a more rigirous theory[10] *the adiabatic approximation* is unacceptable. The shape of the nucleus changes as it passes into an excited state. Thus, in the transition to excited states with $K \neq 0$, the axial symmetry of the nucleus is violated. A change of the nuclear shape in excitation leads to a connection of rotational motion with β-and γ-vibrations.

The reduced electric quadrupole transition probability between the collective states i and f is given by

$$B(E2: i \rightarrow f) = \frac{5}{16\pi} \sum_{mMf} |\langle f| Q_{2m} | i \rangle|^2 \qquad [2.67]$$

where Q_{2m}, for small β- and γ-vibrations about the equilibrium values $\gamma_0 = 0$ and β_o, can be expressed as

$$Q_{2m} = C\, Q_o\, \tau_{2m} \qquad [2.68]$$

while
$$\tau_{2m} = (1 + \mu\xi) \{ (1 - \Gamma^2 z + \ldots) D^2_{mo} + \Gamma \sqrt{Z}(1 - \tfrac{1}{3}\Gamma^2 z) \times (D^2_{m2} + D^2_{m,-2}) \} \qquad [2.69]$$

$$Q_o = \frac{3ZR_o^2\, \beta_o}{\sqrt{5\pi}} ; \quad \xi = \frac{\beta - \beta_o}{\mu \beta_o} ; \quad Z = \frac{\gamma^2}{2\Gamma^2} \qquad [2.70]$$

Calculations give the following results in the units of

$(e^2Q_o^2/16\pi)$ with s and q determined as

$$s = \frac{E2^{+'}}{E2^{+}} \quad \text{and} \quad q = \frac{E0^{+'}}{E2^{+}} \qquad [2.71]$$

(i) For transitions inside the ground rotational band:

$$B(E2:\ I0 \rightarrow I'0) = 5(2I00|I'0)^2(1-\tfrac{1}{s})(1-\frac{2s}{3q^2})^2 \qquad [2.72]$$

where $(2ImM|I'M)$ are the vector addition coefficients.

In particular for the transition from first excited states to ground state, we have

$$B(E2:\ 2^+ \rightarrow 0^+) = (1-\tfrac{1}{s})(1-\frac{2s}{3q^2})^2 \qquad [2.73]$$

(ii) For transitions inside the anomalous band:

$$B(E2:\ I2 \rightarrow I'2) = 5(2I02|I'2)^2(1-\tfrac{2}{s})(1-\frac{3s}{q^2})^2 \qquad [2.74]$$

(iii) For transitions from the anomalous to the ground rotational band:

$$B(E2:\ I2 \rightarrow I'0) = 10(2I\text{-}22|I'0)^2(2s-1)^{-1}(1-\frac{9s}{4q^2})^2$$

$$\times\ [1+\frac{2}{3(2s-1)}\{\frac{(2I02|I'2)}{(2I\text{-}22|I'0)}\left(\tfrac{2}{3}(I-1)I(I+1)(I+2)\right)^{1/2}$$

$$-2-\tfrac{1}{2}(1+(-1)^I)\frac{(2I00|I'0)}{(2I\text{-}22|I'0)}\left(\tfrac{2}{3}(I-1)I(I+1)(I+2)\right)^{1/2}\}] \qquad [2.75]$$

in particular

$$B(E2:\ 2^{+'} \rightarrow 2^+) = \frac{10}{7s}(1+\frac{5}{2s})(1-\frac{9s}{4q^2})^2 \qquad [2.76]$$

$$B(E2:\ 2^{+'} \rightarrow 0^+) = \frac{1}{s}(1-\frac{3}{2s})(1-\frac{9s}{4q^2})^2 \qquad [2.77]$$

2.2(e) <u>Variation of Angular Momentum Projection Approach</u>

This method has been applied by Warke et al.[11] to study the electromagnetic properties of recently observed high spin states in even-even Dy Isotopes. The anomalous behaviour of the calculated spectra is attributed to the sudden change of the nuclear deformation at a certain value of I. The plot of observed moment of inertia as a function of the rotational frequency is very sensitive to the energy level spacings. A slight deviation of tens of keV order pushes the point on the plot far away. The energy spacings and the transition probabilities of E2 cascades between the yrast levels have been successfully explained using many body vibrational projection approach.

2.2(f) <u>Pairing Plus Quadrupole Model</u>

Baranger et al.[12] considered those cases in which the quadrupole force has atleast a slight edge, if not stronger, over the pairing force, since there is a competition through which the nucleus finds its shape. The quadrupole force attempts to deform it while the pairing force tries to keep it spherical. The potential energy of the nucleus has been taken as the function of deformation. If this is not minimum for spherical shape it should occur for an axially symmetric deformation or for an asymmetric one. Kinetic energy is added to the potential energy in Bohr's collective Hamiltonian. The main results obtained for this model can be summarized.

(i) Axial symmetry is preferred in all cases.

(ii) Deformation is maximum in the middle of the shell and tapers off towards both ends, with exact symmetry about the middle.

(iii) For large ratios of quadrupole to pairing forces, the energy gap vanishes and the nucleus attains its maximum axially symmetric deformation. The effect of increasing the pairing force is to reduce the magnitude of deformation, its sign and axial symmetry are preserved during the process.

(iv) During the transition from spherical to deformed shape the phenomenon of double minimum in the potential energy of deformation is observed.

The pairing plus quadrupole model fails to describe the realistic nuclei for weak pairing forces, as the nuclei would then possess an asymmetric deformation.

2.2(g) Pseudo SU(3) Model

Kota[13)] has studied the low-lying spectra of ^{164}Dy, ^{166}Er and ^{168}Yb in the frame work of SU(3) model allowing natural parity neutrons alone to be active. In this model the proton configurations and the unnatural parity neutron configuration have been assumed to be part of the core which carries zero angular momentum. All the nuclei having eight natural parity neutrons in the $n = 4$ shell are studied, but no attempt has been made to study the high spin states.

2.2(h) Sahu's Approach

Sahu et al.[14], convinced by the experimental evidence for the non-axial collective motions in ^{188}Os and ^{188}Pt nuclei, have considered an intrinsic wave function characterized by both the symmetry parameter β and asymmetry parameter γ and angular momentum projections. They have assumed the existance of a static tri-axial shape. The intrinsic calculations have been done in the frame work of Hartree BCS theory employing the pairing plus quadrupole interaction of Baranger and Kumar[12]. The calculated values of level energies, B(E2)s and electromagnetic moments have been found in good agreement with experimental values. Therefore a more rigid tri-axial shape, for these nuclei, has been concluded.

2.2(i) Microscopic Asymmetric Approach

Tanabe et al.[15] have proposed a general formalism for describing microscopically the symmetric and asymmetric rotors in both even and odd mass nuclei. The microscopic operator form of the quantum number specifying the asymmetric rotor state has been provided by the previously proposed macroscopic model, improved by taking into account the D2 symmetry group and the consistency of the approximation. The microscopic theory of the asymmetric rotor is formulated without sacrificing the angular momentum commutation relations. Faessler et al.[16] attempted to apply the particle number projected HFB theory with a linear constraint for I_x to the asymmetric deformation. However, it is impossible to calculate three moments of inertia $\mathcal{J}$ in this scheme, without recourse of all the components of angular momentum and the full D2 symmetry. In their scheme the additional constraints for the new quantum number K, I^2, and H_{rot} are just enough to determine three $\mathcal{J}$'s. At high spins the subsidiary condition for I^2 manifests the existance of the coriolis antipairing effects in the unique parity states i.e. the decoupling effect, which has been observed recently[17].

2.2(j) Toyama's Approach

The collective Hamiltonian for deformed even-even nuclei has been considered to be composed of the vibrational energy term T_{vib}, the rotational energy term T_{rot} and the potential energy term $V(\beta,\gamma)$ by Toyama[18]. The potential term $V(\beta,\gamma)$ is expanded about the centre of oscillation (β_0,γ_0) and anharmonic term of β is introduced. The introduction of the anharmonic term gives a dependence of the moment of inertia not only on I but also on n_β, which is the quantum number of beta vibration. Calculated values of level energies are, with some exceptions, in good agreement with experimental values, and the relative B(E2) values are much improved compared to calculated from Clebsch Gorden Coefficients.

2.2(k) Boson Expansion Microscopic Model

As there existed a strong resemblance between levels in spherical and deformed nuclei, the equations for the coefficients in the expansion of the fermion-pair operators in terms of boson operator[19] are solved exactly to sixth order since the data on level schemes extended the inclusion of higher spin states. A theory has been developed[20,21] which describes the spherical and deformed nuclei and consequently the transitional nuclei on an equal footing.

The nature of the oscillations of spherical nuclei differs essentially from the hydrodynamic model, the nucleons outside the core mainly participate in the oscillations and the polarization of the core arising in the process leads to the renormalization of outer nucleon interaction. This makes it possible to consider only the nucleons in the upper unfilled shell, and the problem is simplified considerably. The microscopic point of view is that the phonon of these oscillations is a strongly correlated (bound) state of two quasi-particles with K = 2 as the total angular momentum. The model predicts the energy of the first excited state and B(E2) value for a transition from this level as well as the dependence of these quantities on the number of outer nucleons. This pair of bound quasi-particles in the nucleus is different from the ideal Bose-excitation (phonon). This difference leads to some additional terms in the Hamiltonian.

2.2(ℓ) <u>New Rigid Asymmetric Rotor Model</u>

DF model has been extended by adding hexadecapole shape components and this extended model[22] has been applied to

$^{184-186}$W, $^{186-192}$Os, $^{192-196}$Pt and ^{198,200}Hg nuclei. The model provides much improved description of the electric quadrupole properties. E2 properties (phases and magnitudes of matrix elements involving the second 2^+ states) have been predicted correctly. An additional degree of freedom β_4 has been included together with β_2 and γ as parameters. This results in fits which are better by two orders of magnitudes than corresponding fits with only β_2 and γ. The values of β_4 and the systematic decrease of $|\beta_4|$ with increase of A, are in excellent agreement with available experimental information. It appears qualitatively that including a β_4 shape component, which has its symmetry about the z-axis has the effect of keeping the nucleus more prolate type as γ increases and the symmetry axis for the quadrupole deformation tends towards the y-axis. It may be noted that $\beta_4/\beta_2 > 0$, leads to the same qualitative features as $\beta_4/\beta_2 < 0$, Therefore it is concluded that the quadrupole and hexadecapole properties of a nucleus are not independent but are coupled and in a detailed model the hexadecapole properties could not be ignored.

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CHAPTER - III

EFFECT OF SPIN ON NUCLEAR SHAPE

3.1 Introduction

Electric quadrupole transition probabilities from 2^+ to 0^+ and from 4^+ to 2^+ states of ground state band of deformed even nuclei have studied by many workers. Some of them[1-5] have used Davydov-Rostovsky[6] (DR) model estimates for these transitions and reported that the factor $F_{DR}[=B(E2)_{exp}/B(E2)_{DR}]$ increases with the increase of non-axiality parameter γ. These workers have scanned the nuclear regions of small γ values and have considered a few transitional nuclei in the rare earth and actinide regions.

Theoretical estimates for deformation parameter β were obtained by calculating the binding energies of the individual nuclei as a function of the nuclear deformation and minimizing the total energy. These estimates were not found consistent with the fact that nuclear deformation decreases sharply as one moves away from closed shell regions for nuclei possessing mass number around 160[7]. This reflects the polarizing effect of particles outside the closed shell. There has also been a discrepancy between experiment and theory related with a fact that the solution of Bardeen, Cooper and Schrieffer (BCS) equation used does not correspond to a definite number of particles in a few cases near the deformation region viz, around Hf and W isotopes[7]. Although Agarwal[1] used rather experimental values of intrinsic quadrupole moment Q_o, but the data of Stelson and Grodzins[8] used by him are now out dated and have changed to a considerable extent. The characteristic properties of asymmetric rotor model (ARM) are not reflected on the systematics of the previous workers[1-5].

This is interesting to refer Hoehn et al.[9], who, with the use of model dependent methods, showed that electric moments involving the first excited state 2^+ and in some cases the second excited state $2^{+\prime}$ differ with those calculated values using pairing plus quadrupole interactions. We have thought worthwhile to consider the nuclear regions which have large γ values, the so called soft nuclei belonging to medium as well as heavy mass regions. For these nuclei it is more promising to attempt the model in the region with γ about 20^o or so, since there is no general theoretical argument to indicate the shape of these nuclei. In our calculations we have used ARM dependent Q_o.

We have done, for the first time Davydov et al.[6,10,11] calculations for $4^+ \rightarrow 2^+$ and $6^+ \rightarrow 4^+$ transitions together with $2^+ \rightarrow 0^+$ transitions for medium mass nuclei, possessing large value of non-axiality parameter γ and for a number of heavy mass nuclei. Medium mass nuclei with γ lying between 15^o to 25^o have been considered.

3.2 Calculation of Transition Probabilities

The experimental transition probabilities in the units of $e^2.b^2$ for the transitions reported here have been calculated by the known relation

$$B(E2:\ I+2 \rightarrow I)_{exp} = \frac{0.0812}{E_\gamma^5 (1+\alpha_T)\,\tau} \qquad [3.1]$$

where E_γ, the energy involved in transition and is expressed in MeV, τ, the meanlife of the excited state is in p.sec. and α_T are the total internal conversion coefficients. These parameters have been taken from Table of Isotopes[12].

$B(E2)_{DF}$ values have been calculated using the relations of Davydov et al.[10,11] and the Table 3.1. The non-axiality parameter γ has been evaluated by the relation[10]

$$\frac{E2^{+'}}{E2^{+}} = \frac{1 + [\ 1 - \frac{8}{9}\,\mathrm{Sin}^2(3\gamma)\ ]^{1/2}}{1 - [\ 1 - \frac{8}{9}\,\mathrm{Sin}^2(3\gamma)\ ]^{1/2}} \qquad [3.2]$$

where $E2^+$ and $E2^{+'}$ are the energies of first and second 2^+ excited states.

$B(E2)_{DR}$ values have been computed by using the relation[6]

$$B(E2:\ I \rightarrow I') = \frac{5e^2 Q_o^{\,2}}{16\pi} (2I00|\,I'0)^2 (1 - \frac{1}{s})(1 - \frac{2s}{3q^2})^2 \qquad [3.3]$$

where $(2I00|\,I'0)$ are Clebsch-Gordan Coefficients in the notations of $(2Jmm'|\,J'm')$ and are taken from reference 6.

TABLE - 3.1

The values of $B(E2: 4^+ \rightarrow 2^+)$ and $B(E2: 6^+ \rightarrow 4^+)$ in the units of $e^2 Q_o^2/16\pi$ as a function of γ as calculated in reference 11.

γ°	0	5	10	15	20	22.5	25	27.5	30
$B(E2: 4^+ \rightarrow 2^+)$	1.429	1.418	1.395	1.377	1.372	1.366	1.365	1.378	1.569
$B(E2: 6^+ \rightarrow 4^+)$	1.573	1.563	1.547	1.562	1.623	1.671	1.703	1.725	1.731

$$s = \frac{E2^{+\prime}}{E2^{+}} \qquad \text{and} \qquad q = \frac{E0^{+\prime}}{E2^{+}} \qquad [3.4]$$

here $E0^{+\prime}$ is the energy of 0^{+} state of beta vibration band. $E2^{+}$, $E2^{+\prime}$ and $E0^{+\prime}$ are taken from Sakai-Rester Table.[13]

3.3 Results and Discussion

Table 3.2 illustrates the experimental, DF and DR values of E(2) transition probabilities for $2^{+}\rightarrow 0^{+}$, $4^{+}\rightarrow 2^{+}$ and $6^{+}\rightarrow 4^{+}$ transitions inside the ground rotational band for heavy and medium mass nuclei. The experimental values have been extracted from reference 12 or have been adapted from reference 14-19. The table reveals that DR model fails to explain almost all the medium mass nuclei and ^{148}Sm, ^{150}Sm, ^{156}Er, and ^{188}Pt nuclei having γ more than 20^{o} in $2^{+}\rightarrow 0^{+}$ and $4^{+}\rightarrow 2^{+}$ transitions, when we test the model according to Kumar's conditions[20] (i.e. 0.5 < enhancement/hinderence factor F < 2). There is no such failure with DF model for $2^{+}\rightarrow 0^{+}$ and $4^{+}\rightarrow 2^{+}$ transitions. However, for $6^{+}\rightarrow 4^{+}$ transitions the hinderence factor F decreases althrough in DR estimates. The figures 3.1 and 3.2 drawn are clear indication of a systematic trend in rigid rotor model while there is no such systematic trend in DR model predictions for $2^{+}\rightarrow 0^{+}$ and $4^{+}\rightarrow 2^{+}$ transitions. This is contrary to the trends observed by previous workers[1-5]. The sudden appearance of a systematic trend in DR model for $6^{+}\rightarrow 4^{+}$ transitions (Fig.3.3) suggests the beginning of a process of shape transition in nuclei at angular momentum I = 6.

TABLE 3.2

B(E2) Values of Gamma-Ray Cascades in units of $e^2.b^2$ Experimental uncertainties are given in brackets.

Nucleus	$B(E2: 2^+ \rightarrow 0^+)$			$B(E2: 4^+ \rightarrow 2^+)$			$B(E2: 6^+ \rightarrow 4^+)$		
	Exptl	DF	DR	Exptl	DF	DR	Exptl.	DF	DR
^{92}Mo	0.018	0.018	-	-	-	-	-	-	-
^{94}Mo	0.039(1)	0.038	0.008	0.067(9)	0.054	0.012	-	-	-
^{96}Mo	0.054(1)	0.053	0.003	0.105(27)	0.075	0.005	-	-	-
^{98}Mo	0.057(3)	0.058	0.016	0.131(12)	0.082	0.023	-	-	-
^{100}Mo	0.123(12)	0.119	0.0005	0.197(15)	0.173	0.0008	-	-	-
^{102}Mo	0.218(22)	0.204	0.040	0.253(42)	0.209	0.057	-	-	-
^{96}Ru	0.053(5)	0.053	-	-	-	-	-	-	-
^{98}Ru	0.080(6)	0.079	0.030	0.108(12)	0.112	0.053	-	-	-
^{100}Ru	0.104(7)	0.102	0.024	0.146(15)	0.147	0.034	-	-	-
^{102}Ru	0.130(9)	0.128	0.009	0.201(38)	0.183	0.013	-	-	-
^{104}Ru	0.164(12)	0.163	0.060	0.217(4)	0.231	0.086	-	-	-

Contd...54....

^{108}Ru	0.185(17)	0.185	0.105	-	-	-	-	-	-
^{102}Pd	0.09(1)	0.090	0.047	0.147(9)	0.130	0.067	-	-	-
^{104}Pd	0.104(4)	0.103	0.045	0.152(9)	0.148	0.065	-	-	-
^{106}Pd	0.124(8)	0.125	0.033	0.220(30)	0.174	0.047	-	-	-
^{108}Pd	0.146(10)	0.147	0.044	0.280(40)	0.205	0.063	-	-	-
^{110}Pd	0.172(12)	0.171	0.055	0.310(40)	0.240	0.079	-	-	-
^{106}Cd	0.077(7)	0.079	-	0.136(15)	0.114	-	-	-	-
^{108}Cd	0.081(7)	0.082	0.027	0.120(15)	0.118	0.038	-	-	-
^{110}Cd	0.085(1)	0.082	0.023	-	-	-	-	-	-
^{112}Cd	0.108(8)	0.108	0.023	0.194(19)	0.151	0.033	-	-	-
^{114}Cd	0.117(8)	0.116	0.027	0.211(36)	0.163	0.166	-	-	-
^{116}Cd	0.120(8)	0.117	0.038	0.191(36)	0.167	0.055	-	-	-
^{120}Te	0.105(2)	0.105	0.010	-	-	-	-	-	-
^{122}Te	0.132(2)	0.132	0.055	0.179	0.186	0.079	-	-	-
^{124}Te	0.114(2)	0.117	0.045	-	-	-	-	-	-
^{126}Te	0.095(2)	0.093	0.034	-	-	-	-	-	-
^{128}Te	0.078(7)	0.077	0.026	-	-	-	-	-	-
^{130}Te	0.058(1)	0.058	-	-	-	-	-	-	-

Contd 55......

^{120}Xe	0.189(24)	0.179	-	0.385(82)	0.258	-	-	-	-
^{122}Xe	0.229(2)	0.217	-	0.325(51)	0.246	-	-	-	-
^{124}Xe	0.180(16)	0.172	-	-	-	-	-	-	-
^{126}Xe	0.154(1)	0.149	-	-	-	-	-	-	-
^{128}Xe	0.137(11)	0.133	0.058	-	-	-	-	-	-
^{130}Xe	0.198(15)	0.192	0.079	-	-	-	-	-	-
^{132}Xe	0.087(7)	0.087	-	-	-	-	-	-	-
^{130}Ba	0.248(76)	0.235	-	-	-	-	-	-	-
^{132}Ba	0.144(32)	0.139	0.058	-	-	-	-	-	-
^{134}Ba	0.133(59	0.133	0.046	0.214	0.202	0.094	-	-	-
^{136}Ba	0.082	0.082	0.017	-	-	-	-	-	-
^{138}Ba	0.042(2)	0.042	-	-	-	-	-	-	-
^{132}Ce	0.329(43)	0.320	-	0.43 (11)	0.451	-	-	-	-
^{134}Ce	0.214(14)	0.206	-	0.169(34)	0.290	-	-	-	-
^{142}Nd	0.081	0.081	0.007	-	-	-	-	-	-
^{144}Nd	0.101	0.098	0.039	0.030(5)	0.138	0.055	-	-	-
^{146}Nd	0.146(14)	0.137	0.072	-	-	-	-	-	-
^{148}Nd	0.274(7)	0.256	0.104	-	-	-	-	-	-

Contd..56......

^{150}Nd	0.640(20)	0.610	0.360	0.851(69)	0.883	0.511	1.00(7)	0.998	0.56
^{148}Sm	0.141(5)	0.139	0.029	0.248	0.201	0.041	-	-	-
^{150}Sm	0.274(6)	0.256	0.049	0.374(7)	0.388	0.070	-	-	-
^{152}Sm	0.670(15)	0.657	0.430	1.030(10)	0.950	0.610	1.16(2)	1.07	0.67
^{154}Sm	0.922(40)	0.911	0.760	1.180(30)	1.306	1.090	1.31(4)	1.45	1.19
^{152}Gd	0.394(26)	0.368	0.029	-	-	-	-	-	-
^{154}Gd	0.770(16)	0.760	0.520	1.180(40)	1.103	0.750	1.36(3)	1.25	0.82
^{156}Gd	0.914(10)	0.918	0.740	1.290(20)	1.321	1.060	1.47(3)	1.47	1.16
^{158}Gd	0.994(10)	0.948	0.850	1.37 (11)	1.362	1.210	-	-	-
^{160}Gd	1.030(10)	1.015	-	-	-	-	-	-	-
^{156}Dy	0.753(46)	0.712	0.428	-	-	-	-	-	-
^{158}Dy	0.980(76)	0.969	0.770	1.29 (16)	1.402	1.100	-	-	-
^{160}Dy	0.998(60)	0.961	0.847	1.49(13)	1.414	1.210	1.23(6)	1.58	1.33
^{162}Dy	1.089(30)	1.061	0.917	1.530(90)	1.540	1.310	1.38(6)	1.72	1.44
^{164}Dy	1.140	1.095	-	1.570	1.610	-	1.67(6)	1.80	1.28
^{156}Er	0.330(20)	0.312	0.119	0.540	0.554	0.170	1.02(89)	0.55	0.19
^{158}Er	0.560(30)	0.520	0.294	0.880(40)	0.769	0.420	1.15(72)	0.90	0.46
^{160}Er	0.840(40)	0.795	0.595	1.170(60)	1.156	0.850	1.36(12)	1.31	0.93

^{162}Er	0.976(49)	0.933	0.778	-	-	-	-	-	-
^{164}Er	1.150(70)	1.135	0.959	1.40 (13)	1.641	1.370	-	-	-
^{166}Er	1.122(40)	1.076	0.967	1.69(10)	1.591	1.380	1.60	1.78	1.52
^{168}Er	1.170(40)	1.150	1.000	1.67 (10)	1.658	1.430	-	-	-
^{170}Er	1.185(30)	1.160	0.960	1.550(40)	1.672	1.370	-	-	-
^{160}Yb	0.464	-	-	-	-	-	0.86(9)	0.77	0.39
^{162}Yb	0.746	-	-	-	-	-	1.09(21)	1.18	0.64
^{164}Yb	0.930(35)	0.880	0.680	1.370(50)	1.280	0.970	1.47(5)	1.45	1.06
^{166}Yb	1.06 (15)	1.010	0.810	1.470(50)	1.467	1.160	1.57(6)	1.65	1.27
^{168}Yb	1.080(50)	1.050	0.906	-	-	-	-	-	-
^{170}Yb	1.040(20)	1.010	0.860	-	-	-	-	-	-
^{172}Yb	1.186(50)	1.160	0.970	1.81 (43)	1.669	1.380	1.76(9)	1.85	1.52
^{174}Yb	1.148(56)	1.130	1.010	1.610(40)	1.617	1.450	1.74(10)	1.79	1.59
^{176}Yb	1.050	1.030	-	1.52 (14)	1.481	-	1.50(19)	1.64	1.27
^{166}Hf	0.86	-	-	-	-	-	1.11(16)	1.36	0.68
^{168}Hf	0.836	-	-	-	-	-	1.28(13)	2.13	0.86
^{170}Hf	1.26(27)	-	-	-	-	-	1.47(12)	1.87	-
^{172}Hf	0.928(60)	0.890	0.700	-	-	-	-	-	-

Contd.58.......

${}^{174}Hf$	0.935(44)	0.932	0.710	-	-	-	-	-	-
${}^{176}Hf$	1.215(35)	1.200	1.020	-	-	-	-	-	-
${}^{178}Hf$	0.998	0.980	0.850	-	-	-	-	-	-
${}^{180}Hf$	0.974(13)	0.945	0.550	1.32 (17)	1.357	0.788	-	-	-
${}^{180}W$	0.863(21)	0.822	-	-	-	-	-	-	-
${}^{182}W$	1.029(11)	1.020	0.850	1.235(90)	1.465	1.210	1.67(17)	1.92	1.54
${}^{184}W$	0.667(12)	0.660	0.540	0.954(8)	0.960	0.770	1.68(17)	1.70	1.26
${}^{186}W$	0.669(9)	0.660	0.460	0.892	0.962	0.660	1.89(29)	1.59	1.10
${}^{182}Os$	1.702(9)	0.665	-	-	-	-	-	-	-
${}^{184}Os$	0.569(40)	0.540	0.430	0.84 (26)	0.785	0.620	-	-	-
${}^{186}Os$	0.630(6)	0.630	0.450	0.990	0.917	0.650	1.17	1.05	0.71
${}^{188}Os$	0.568(6)	0.580	0.370	0.790	0.847	0.530	1.16	0.99	0.59
${}^{190}Os$	0.496(4)	0.510	0.270	0.620	0.743	0.390	1.06	0.90	0.42
${}^{192}Os$	0.420(4)	0.440	-	0.540(70)	0.628	-	0.87 (14)	0.78	0.33
${}^{184}Pt$	1.070(30)	0.990	0.400	-	-	-	-	-	-
${}^{186}Pt$	0.710(30)	0.660	0.200	-	-	-	-	-	-
${}^{188}Pt$	0.520(80)	0.510	-	-	-	-	-	-	-

Contd. 59........

^{190}Pt	0.52 (19)	0.520	0.190	-	-	-	-	-	-
^{192}Pt	-	-	-	-	-	-	0.47(18)	0.59	0.20
^{228}Th	1.41 (12)	1.370	1.190	1.720(70)	1.960	-	-	-	-
^{230}Th	1.60 (40)	1.570	1.290	2.290(70)	2.260	1.840	-	-	-
^{232}Th	1.850 (80)	1.820	1.570	2.67 (21)	2.610	2.240	3.09(2)	2.89	2.53
^{232}U	1.98 (24)	1.930	1.660	-	-	-	-	-	-
^{234}U	2.030(90)	1.980	1.780	-	-	-	-	-	-
^{236}U	2.320(80)	2.270	2.060	3.52(19)	3.240	-	3.52(20)	3.66	3.24
^{238}U	2.380(50)	2.330	2.110	-	-	-	-	-	-
^{238}Pu	2.520(70)	2.460	2.240	-	-	-	-	-	-
^{240}Pu	2.530(70)	2.470	2.230	-	-	-	-	-	-
^{242}Pu	2.68 (11)	2.630	2.380	-	-	-	-	-	-
^{244}Pu	2.770(70)	2.710	-	-	-	-	-	-	-
^{246}Cm	3.010(90)	2.950	2.760	-	-	-	-	-	-

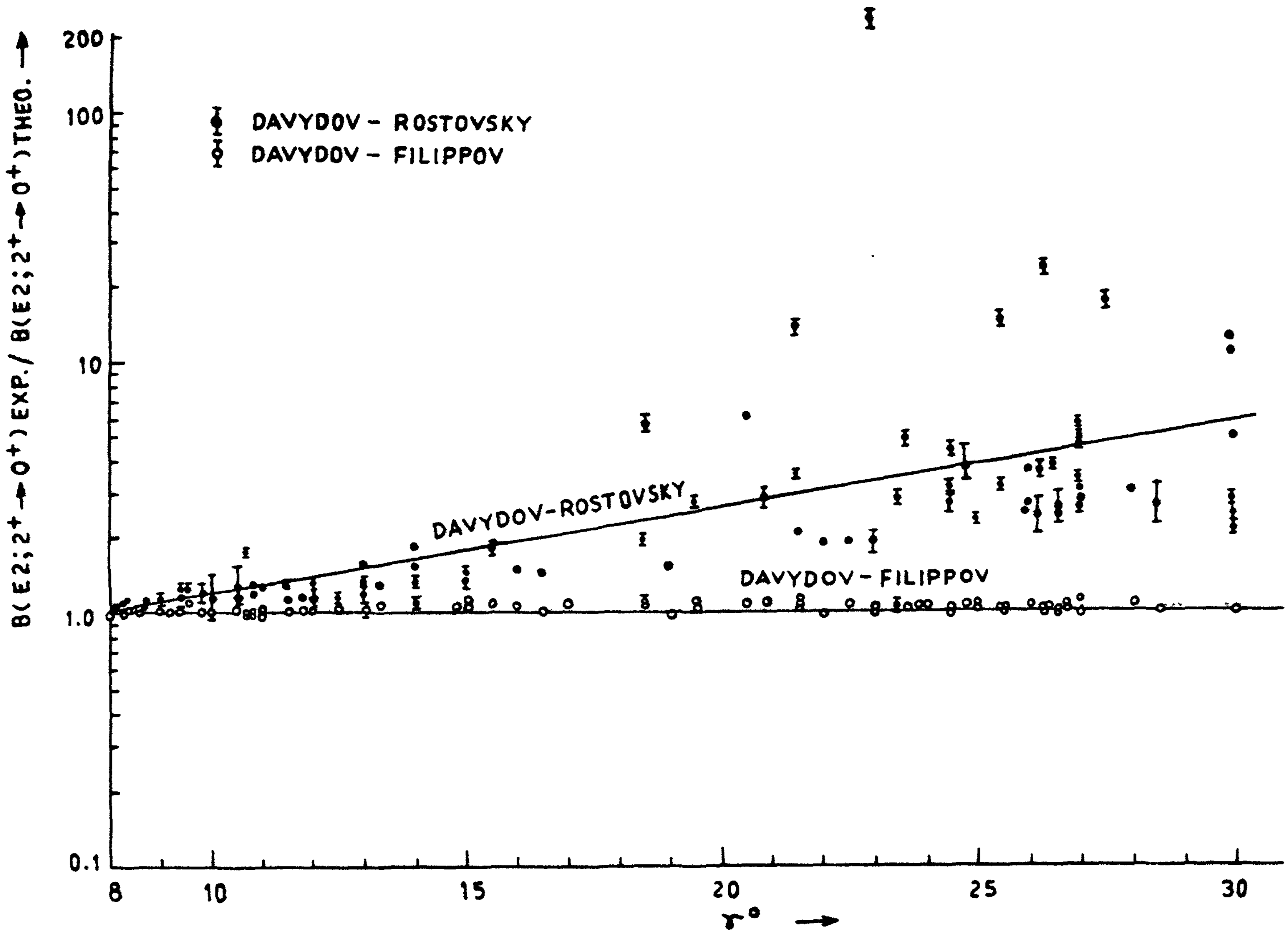

Fig. 3.1 Plot of $B(E2;2^+ \rightarrow 0^+)_{exp.} / B(E2;2^+ \rightarrow 0^+)_{theo.}$ versus γ.

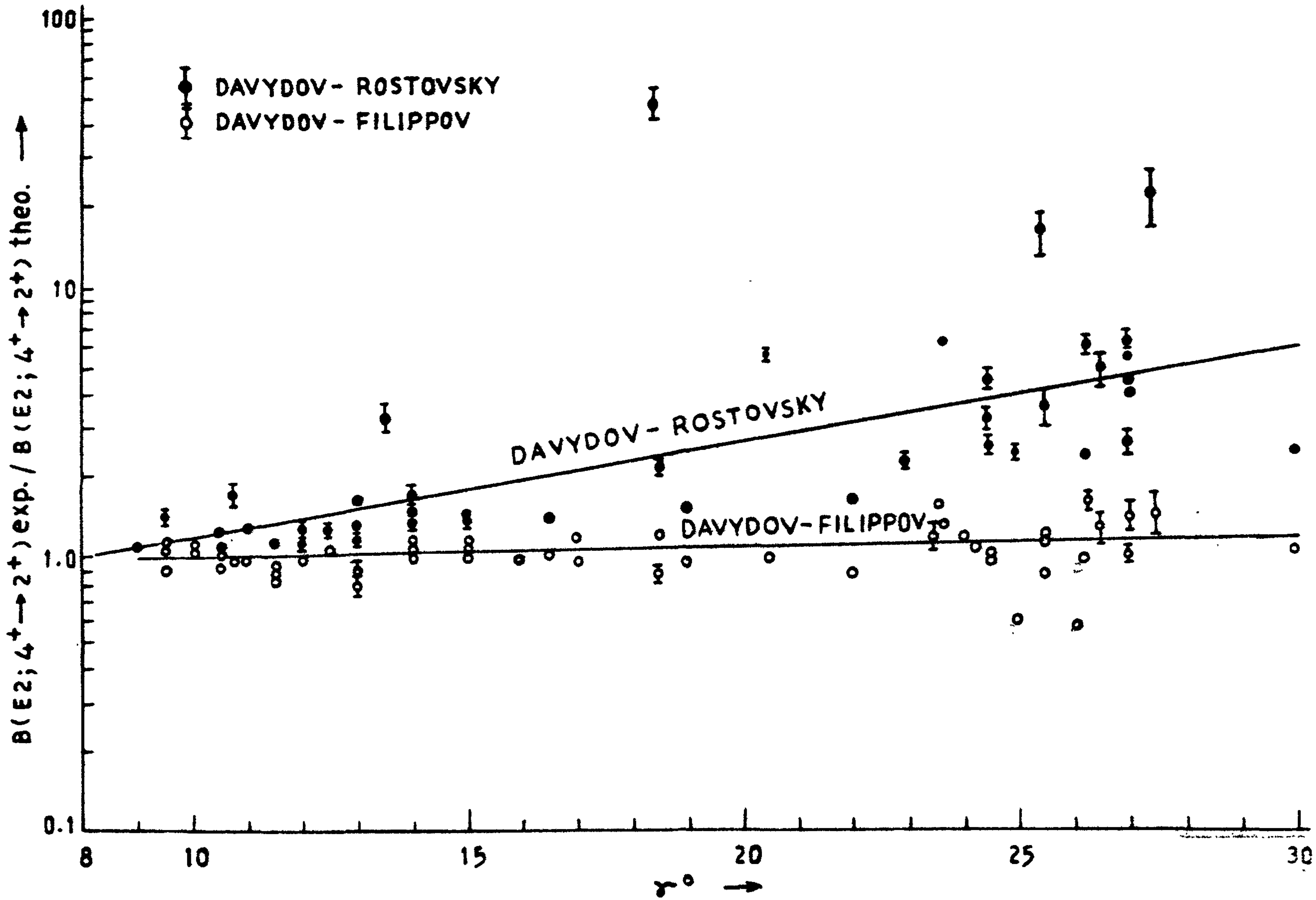

Fig. 3.2 Plot of $B(E2;4^+\rightarrow 2^+)$ exp. / $B(E2;4^+\rightarrow 2^+)$ Theo. versus γ.

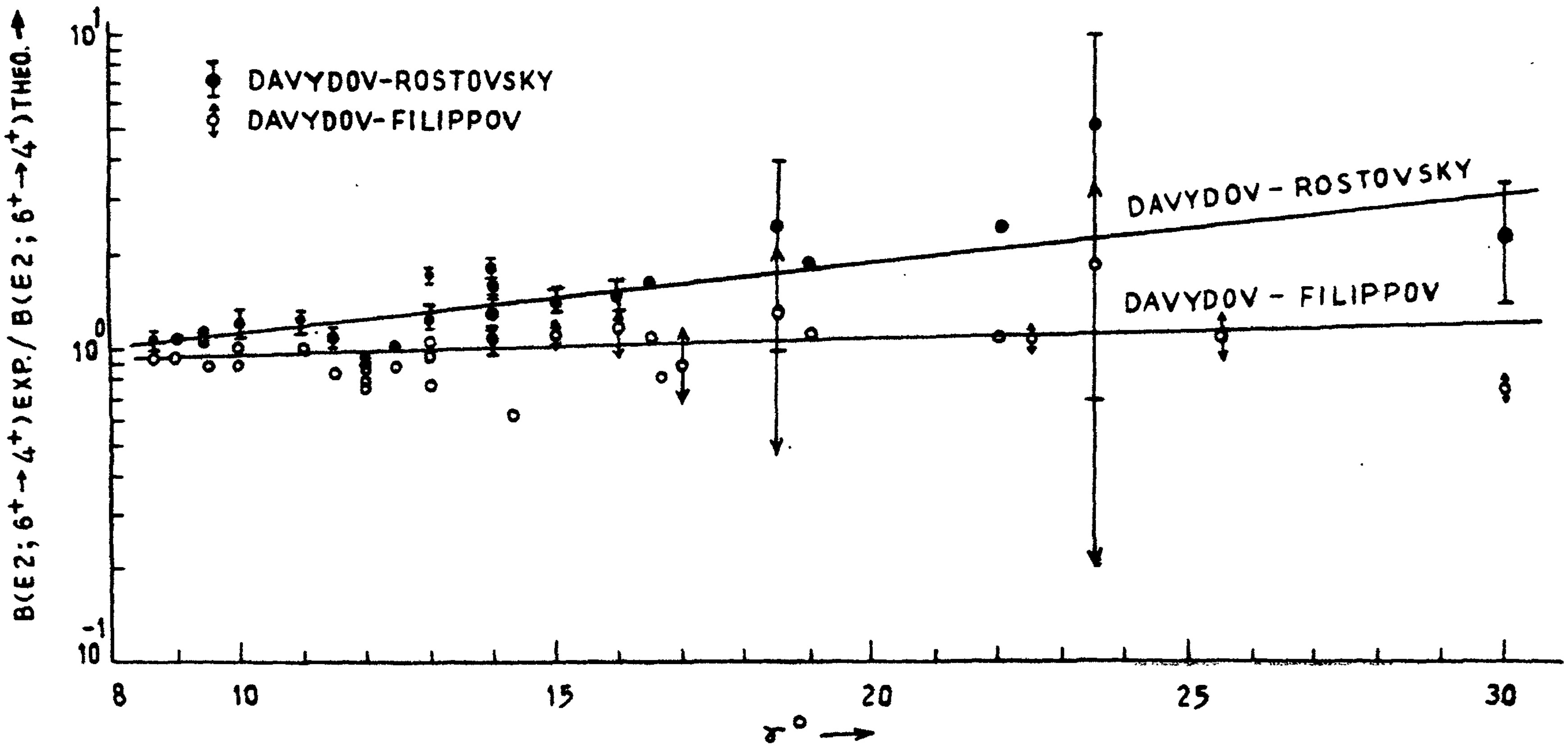

Fig. 3.3 Plot of $B(E2; 6^+ \to 4^+)$ EXP. / $B(E2; 6^+ \to 4^+)$ THEO. versus γ.

This surprising change is observed for a number of nuclei lying on or near the trends for $6^+ \rightarrow 4^+$ transitions with DF and DR predictions. The nature of the agreement observed for $2^+ \rightarrow 0^+$ and $4^+ \rightarrow 2^+$ transitions is found to be reversed in both the models. The factor F which shows almost a uniform sign in $2^+ \rightarrow 0^+$ and $4^+ \rightarrow 2^+$ transitions irrespective of value of γ in DF model starts giving variable sign for $6^+ \rightarrow 4^+$ transitions which is also γ dependent. No such discrepancy has been observed in DR model.

Although the adiabatic approximation (β and γ constant) used by Davydov-Filippov is unacceptable in a more rigorous theory, but in general, violation of the adiabatic condition during rotation can be due to coriolis interaction between the angular momentum and single nucleon states and also due to a centrifugal stretching. The centrifugal stretching leads to a change in the shape of the nucleus during rotation and hence to a change in the moment of inertia of the nucleus. The mean values of the total angular momentum projections of the outer nucleons on the arbitrary direction in the nuclei are zero in even non-spherical nucleus in the ground state (with respect to the other nucleons). Transition to an excited single nucleon state, on the other hand, requires a large energy and therefore there is no coriolis interaction.

According to general belief, the shape of the nucleus changes as it passes into an excited state and this change of nuclear shape in excitation leads to some interaction of rotational motion with β-and γ-vibrations. Such assumptions of DR theory lead to a simultaneous descriptions of ground band, beta band and gamma band. But the failure of DR model predictions upto spin 4 confirm that the adiabatic approximation is satisfactory only if one considers the excited states of the 2^+ and 4^+ ground band, the first excited states of the anomalous rotational band and the probabilities for transitions between them[21). The process of shape transition commences at I = 6.

3.4 Conclusion

It is inferred that the assumption of rigid tri-axial shape with fixed shape parameter β and γ is an excellent approximation to the actual nuclear wave functions only upto spin 4 and thereafter the nucleus begins to get rid of rigid shape and acquires β- and γ-freedom. Our study lends support to the assumption of tri-axial shape of Davydov et al. and suggests a limit at which the rigidity of the nucleus is unacceptable. This conclusion definitely differs with Turner et al.[22) view point of abrupt phase transition of the rotational mode from axially symmetric to asymmetric shape at I = 8.

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CHAPTER - IV

STUDY OF SAMARIUM ISOTOPES ($^{148,150,152,154}Sm$)

4.1 Introduction :

Study of Samarium isotopes has been a challanging theoretical problem since they lie in the range from near spherical to well deformed shape. ^{148}Sm nucleus is believed to be basically spherical while ^{154}Sm is thought to be a well deformed nucleus and $^{150,152}Sm$ are considered transitional ones. Various approaches[1-3] have been applied in the past but noneof them was found to be fully successful in explaining the known B(E2) values and B(E2) branching ratios of inter and intra band transitions. Since Asymmetric rotor model of Davydov-Filippov[4] has been proved useful particularly for nuclei in the transition region, it has been thought worthwhile to study $^{148,150,152,154}Sm$ nuclei in the frame work of Asymmetric Rotor Model (ARM). The comparison of phenomenological[4] model results with those of microscopic model calculations of Kumar[1] and Tamura et al.[3] has been done. Though it is not proper to compare on an equal footing, the phenomenological model with microscopic models, even then, it is done to emphasize the better fit achieved by DF model over the other existing models.

Recently Gupta et al.[5] have evaluated the B(E2) values for the transitions depopulating 2^+ state of gamma vibrational band and for inter rotational band in deformed even nuclei. Their results have contradicted Zawischa et al[6] view point in general and confirmed that low-lying $K^{\pi} = 2^{+}$ resonances are classical gamma vibrations for Samarium nuclei. The anomalous ^{152}Sm nucleus, having neutron number 90, lies in the vicinity of the experimental trend of $B(E2: 2^{+\prime} \rightarrow 2^{+}/0^{+})$ vs non-axiality parameter γ, which is fully endorsed in the DF trend[5]. Puri et al.[7] studied the properties of $2^{+\prime}$ level in $150 < A < 190$ region nuclei and observed that experimental δ values ($E2/M_1$ mixing ratio, which provides rather a sensitive test of nuclear wave functions) favoured the non-axial model calculations of Davydov et al.[4] over the pairing force model of Greiner[8] or the coriolis interaction model of Bes et al.[9]

Further the reduced electric quadrupole transition probabilities in $e^2.b^2$ units from first 2^{+} state are changing very rapidly, from 0.141 to 0.274 between ^{148}Sm and ^{150}Sm and from 0.274 to 0.667 between ^{150}Sm and ^{152}Sm. This indicates a rapid change in the average γ values as can be seen from the expression for the quadrupole moment of the first 2^{+} state of a tri-axial nucleus[4]

$$Q_2 = \frac{3ZR^2\beta}{\sqrt{5\pi}} \cdot \frac{6\cos(3\gamma)}{7[9-8\sin^2(3\gamma)]^{1/2}} \qquad [4.1]$$

Another point in favour of ARM is that there exists a relation

$$E2^{+\prime} + E2^{+} \approx E3^{+} \qquad [4.2]$$

for the four isotopes of Samarium (Table 4.1).

The non-axiality parameter γ has been calculated using all the existing methods which have been explained in the next article.

4.2 Methods For Calculating γ :

The following four methods have been used for evaluating the non-axiality parameter γ for the Samarium isotopes.

(a) From the energy ratio[4] $E2^{+'}/E2^{+}$

(b) From the energy ratio[10] $E4^{+}/E2^{+}$

(c) From the energy ratio[11] $E2^{+'}/E4^{+}$ or $E6^{+}$

(d) From the values[12] of $E2^{+}$, $B(E2: 2^{+} \rightarrow 0^{+})$ and model dependent Q_o.

Method (a) is most widely used for calculating γ. But when method (a) gave anomalous description of gamma band energies pertaining to ^{154}Gd nucleus, method (b) was suggested by Varshni et al.[10] Moreover calculations based on method (a) put non-DF nuclei in DF region.

During the last decade many workers[5,7,11,13,14] followed method (b) and commented on B(E2) values of even deformed nuclei in the rare earth and actinide regions. Puri et al.[7] using method (b) inferred the lead of DR[15] model over DF[4] model for B(E2) ratios of the cascades to cross-over transitions from the 2^{+} vibrational band. This was against the fact since the inclusion of the coupling of rotation with β-vibrations worsened the agreement with experimental values[5]. An

TABLE - 4.1

List of $E2^+$, $E2^{+'}$, $E3^+$, $B(E2: 2^+ \rightarrow 0^+)$, $B(E2: 2^{+'} \rightarrow 0^+)$, $e^2Q_o^2/16\pi$, $\beta A^{2/3}$ and non-axiality parameter γ for $^{148,150,152,154}Sm$ Nuclei.

Nucleus	$E2^+$ (keV)	$E2^{+'}$ (keV)	$E3^+$ (keV)	$B(E2:2^+ \rightarrow 0^+)$ ($e^2.b^2$)	$B(E2:2^{+'} \rightarrow 0^+)$ ($e^2.b^2$)	$e^2Q_o^2/16\pi$ ($e^2.b^2$)	Non-axiality parameter γ (A)	(B)	(C)	(D)	$\lvert\beta A^{2/3}\rvert$
^{148}Sm	550.1	1453.6	1902.9	0.141(5)	0.0061(10)	0.147(60)	23.7	-	19.7	12.0	4.04
^{150}Sm	333.9	1193.81	1504.53	0.274(6)	0.0088(20)	0.283(8)	20.5	-	17.5	20.25	5.58
^{152}Sm	121.77	1083.79	1233.8	0.670(15)	0.0163(11)	0.686(16)	13.0	22.0	12.5	14.5	8.68
^{154}Sm	82.05	1522.45	1540.0	0.922(40)	0.013(3)	0.935(43)	9.5	15.5	9.5	10.0	10.14

explanation to this can be sought on analysing the work of Varshni et al.[10]. The method (b) gave γ few degrees larger than by method (a) at nearly 15°. This enhancement in γ values may be thought as equivalent to introducing the Bohr-Mottelson vibration rotation interaction correction (BMVRIC) of the form $-bI^2(I+1)^2$ in the energy values. Therefore if γ are taken from ground state rotational band energies, then the energies of the remaining levels of this band may be predicted to an accuracy of 1% but the energies of $2^{+\prime}$, 3^{+} and $4^{+\prime}$..... levels can not be predicted with any precision(Fig. 4.1 and 4.2). Another objectionable outcome of method (b) is that it is not applicable in ^{150}Sm and keeps this nucleus out of DF range. This is also against the fact that it can be the most suitable nucleus for ARM, being transitional.

For ^{152}Sm nucleus method (b) gives $\gamma = 22^{\circ}$ and method (a) gives $\gamma = 13^{\circ}$. The enhancement in γ amounting to 9°(77%) can not be justified as equivalent to rotation vibration interaction term, which can be of few degrees only. The method (b) can be of utility if one is confined to ground state energy levels. Therefore the use of such γ for evaluating B(E2) absolute values and B(E2) branching ratios as done by some workers[7,13] has no justification.

The method (c) of deducing non-axiality parameter γ from the energy ratio $E2^{+\prime}/E4^{+}$ or $E6^{+}$ suggested by Meyertervehn[11)

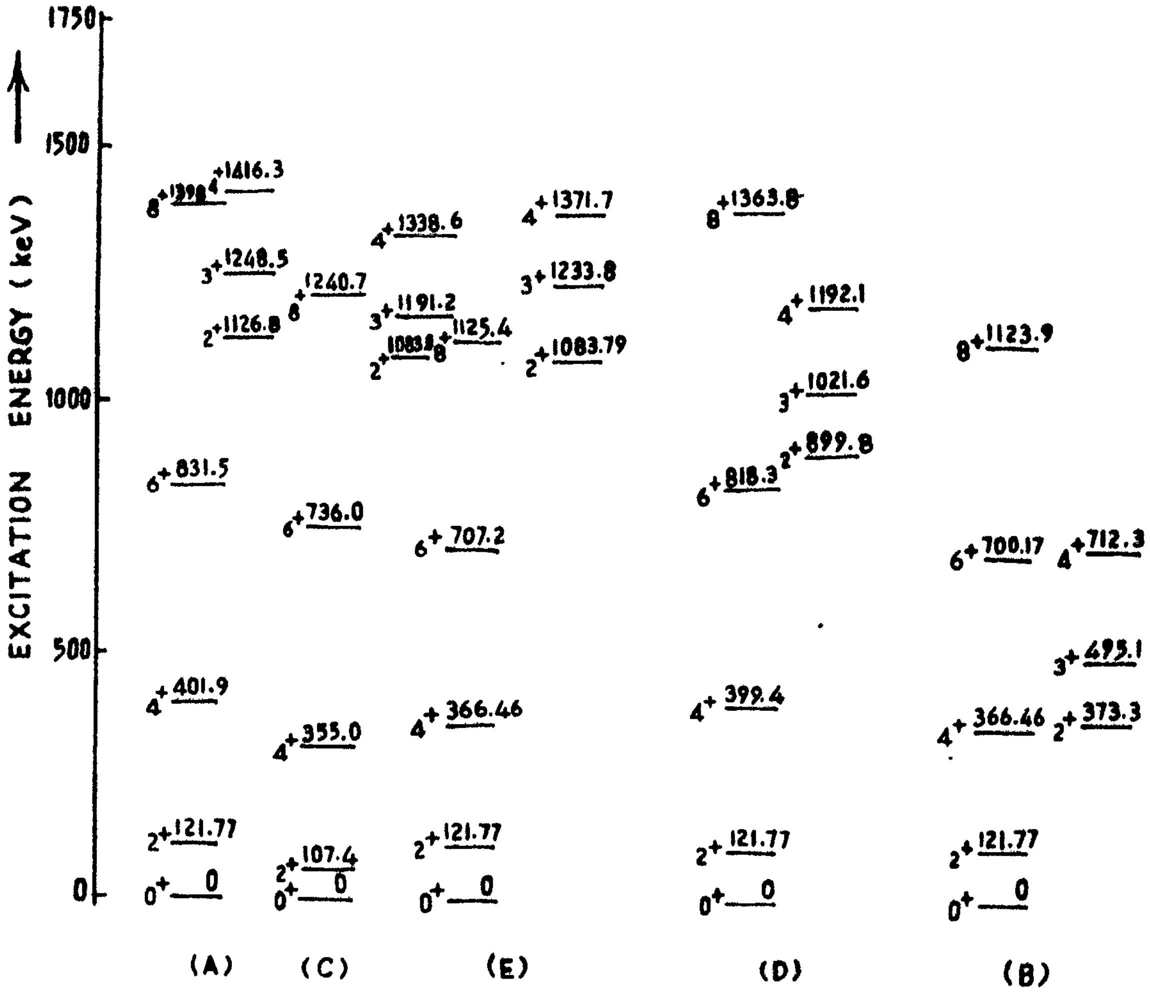

Fig. 4.1 Energy spectrum for ^{152}Sm nucleus.

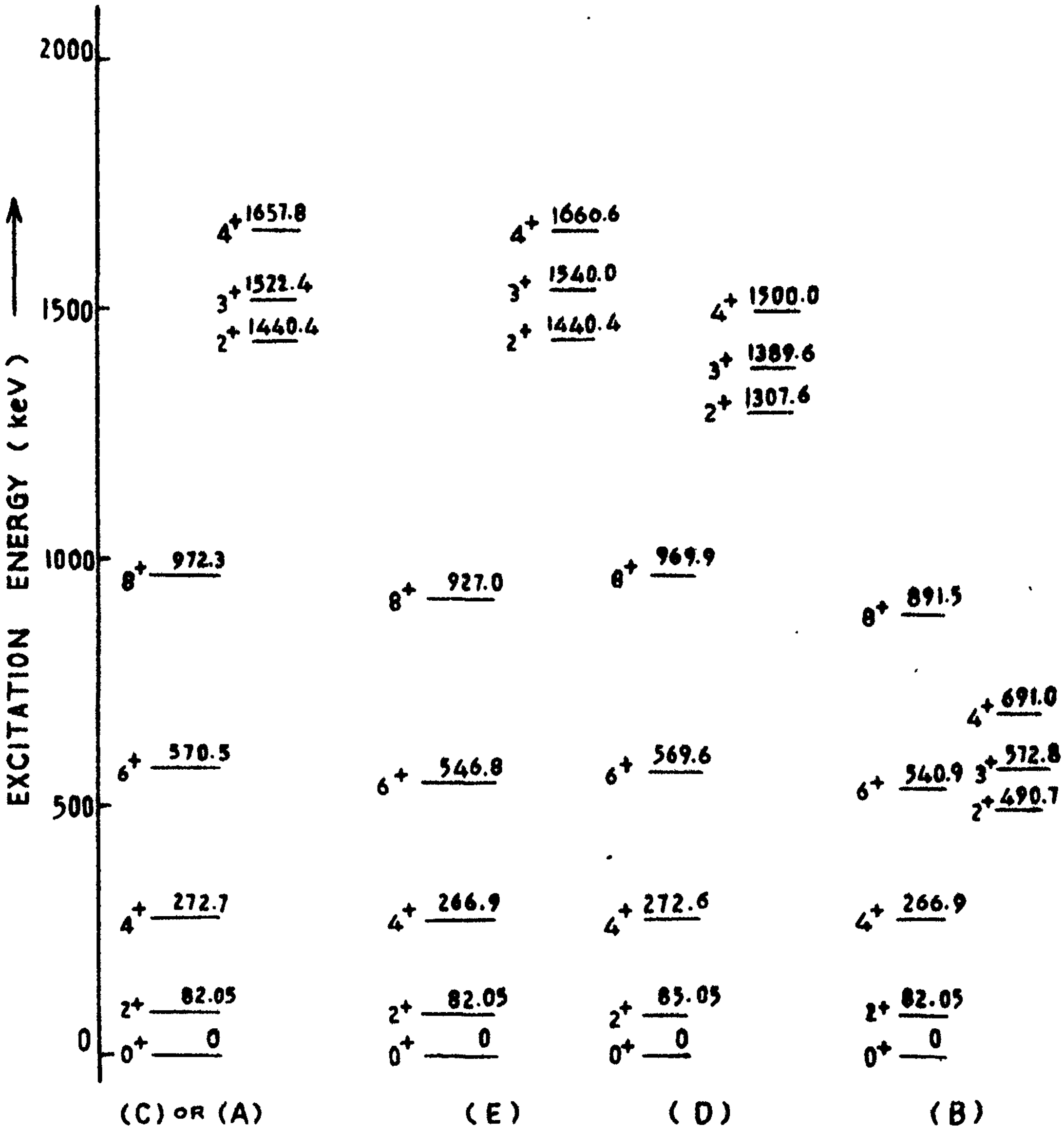

Fig. 4.2 Energy spectrum for ^{154}Sm nucleus.

has been commented unreliable by Baker et al.[16].

Since none of the methods described above for the determination of non-axiality parameter γ, was found capable of explaining the observed electric properties like B(E2) absolute values and B(E2) branching ratios and the energy spectra of four isotopes of Samarium, hence a new method[12] is employed together with other existing methods (a),(b) and (c). For a fixed value of β, violation of axial symmetry of the nucleus leads to an increase of the energy of the levels belonging to the axial nucleus in DF model. The increase of level energy corresponds to a decrease of effective moment of inertia of the nucleus. For the first excited state of spin 2,

$$E2^{+} = \frac{6\hbar^2}{2\mathcal{J}_o}\left[\frac{9 - \{81 - 72\,\mathrm{Sin}^2(3\gamma)\}^{1/2}}{4.\,\mathrm{Sin}^2(3\gamma)}\right] \qquad [4.3]$$

where the inertial parameter, according to general empirical rule of Grodzins[17] is

$$\frac{6\hbar^2}{2\mathcal{J}_o} \approx 1224\, A^{-7/3}\, \beta^{-2} \qquad [4.4]$$

The reduced transition probabilities $B(E2\colon I_i \rightarrow I_f)$ are functions of Q_o and are expressed as

$$B(E2\colon 2^{+} \rightarrow 0^{+}) = \frac{e^2 Q_o^{\,2}}{32\pi}\left[1 + \frac{3 - 2\,\mathrm{Sin}^2(3\gamma)}{\{9 - 8\,\mathrm{Sin}^2(3\gamma)\}^{1/2}}\right] \qquad [4.5]$$

and $$B(E2\colon 2^{+\prime} \rightarrow 0^{+}) = \frac{e^2 Q_o^{\,2}}{32\pi}\left[1 - \frac{3 - 2\,\mathrm{Sin}^2(3\gamma)}{\{9 - 8\,\mathrm{Sin}^2(3\gamma)\}^{1/2}}\right] \qquad [4.6]$$

If the deformation can be characterized by a single parameter β, then Q_o is approximately given[18] by

$$Q_o = \frac{3\ ZR^2\ \beta}{\sqrt{5\pi}} \qquad [4.7]$$

Equation (4.3) allow us to express γ interms of $E2^+$, $B(E2:\ 2^+ \rightarrow 0^+)$, Z and A. Hence mass number A and charge Z of nucleus also play a role in deciding the shape of the nucleus while describing asymmetric parameter γ. The parameters $\beta A^{2/3}$ and γ for the four isotopes of Samarium are listed in Table 4.1 together with other input quantities.

According to classical approximation to the dynamics of triaxial core which prefers rotations about the axis with largest moment of inertia in order to minimize the rotational energy, we have the following classification.

(i) $|\beta A^{2/3}| < 4$, for vibrational nuclei.

(ii) $|\beta A^{2/3}| > 7$, for well deformed nuclei.

(iii) $4 < |\beta A^{2/3}| < 7$, for transitional nuclei.

From Table 4.1 we see that ^{148}Sm nucleus for which $|\beta A^{2/3}|$ is equal to 4.02 is almost vibrational nucleus. ^{150}Sm nucleus for which $|\beta A^{2/3}| = 5.58$ is transitional, with $\gamma = 20.25^o$ ^{152}Sm for which $|\beta A^{2/3}| = 8.68$ is near to well deformed with $\gamma = 14.5^o$ and ^{154}Sm for which $|\beta A^{2/3}| = 10.14$ is well deformed with $\gamma = 10^o$. Thus the approach (d) describes the Samarium nuclei strictly according to established facts.

As the energy levels are not a very good probe of the nuclear shape because they are rather sensitive to softness and are sensitive to inertial parameter whose γ-dependence corresponds to irrotational flow in addition to other effects such as coriolis antipairing (CAP), the calculation of B(E2) values which are more sensitive to γ have been carried out for some useful conclusions.

4.3 Results and Discussion :

4.3(a) Energy Levels:

Figures 4.1 and 4.2 show the energy spectra of ^{152,154}Sm nuclei. In column (E), the experimental energy levels taken from Table of Sakai and Rester[18] are listed. Column (A), (D) and (B) show the theoretical ARM energy levels calculated by using the non-axiality parameter γ derived from methods (a), (d) and (b) respectively. We note that the calculated values for $2^{+\prime}$, 3^+ and $4^{+\prime}$ levels in column (B) are too much lower than those of experimental values. The agreement of energy values for gamma band between column (A) and (E) is best but for column (A) the input parameters are $E2^+$ and $E2^{+\prime}$ respectively. Column (D) also competes well with column (E) where the inputs are $E2^+$ and B(E2: $2^+ \rightarrow 0^+$). This can be qualitatively argued as the minimum finite value of γ for stable deformation which can not be small since the frequency of vibration and rotation would then be comparable and seperation into rotational and vibrational levels would be meaningless. For $\gamma < 15^\circ$, the non-axiality parameter deserves to be enhanced by a degree or two equivalent to BMVRIC. Probably this is the reason why column (D) results are better than those of column (A) and (C) for ground state levels. The results of column (B) are discouraging. Again the reason that the value of γ should be lowered for $\gamma > 21.5$ by few degrees gets support from DF energy diagram (Fig. 2.2 of Chapter II) which reads $E2^{+\prime} = E4^{+}$ at $\gamma = 21.5^{\circ}$, while experimental $E2^{+\prime}$ (= 1.1938 MeV) is much higher to experimental $E4^{+}$ (=0.3665 MeV) for ^{152}Sm. This fact rejects the value of $\gamma = 22^{\circ}$ used in column (B).

Fig. 4.3 shows tha energy spectrum of ^{150}Sm nucleus. In column (E) the energy levels taken from Table of Sakai and Rester[18] are listed. Column (A), (D) and (C) display the theoretical ARM energy levels derived on putting γ calculated from methods (a), (d) and (c) respectively. The discrepancy between calculated and observed energy levels were attempted to be explained by Mallmann[19] taking into account the interaction of rotation with β-vibrations. This interaction can be expressed as

$$E_b(^nI, \gamma,\beta) = A[\varepsilon(^nI,\gamma) - b\{\varepsilon(^nI,\gamma)\}^{1/2}] \qquad [4.8]$$

where b is constant, $\varepsilon(^nI,\gamma)$ are eigen values in units of $A(= \hbar^2/4B\beta^2)$ for DF model without interaction. But such correction can not be applied in the results of column (C) for ^{150}Sm, since the discrepancies between calculated and

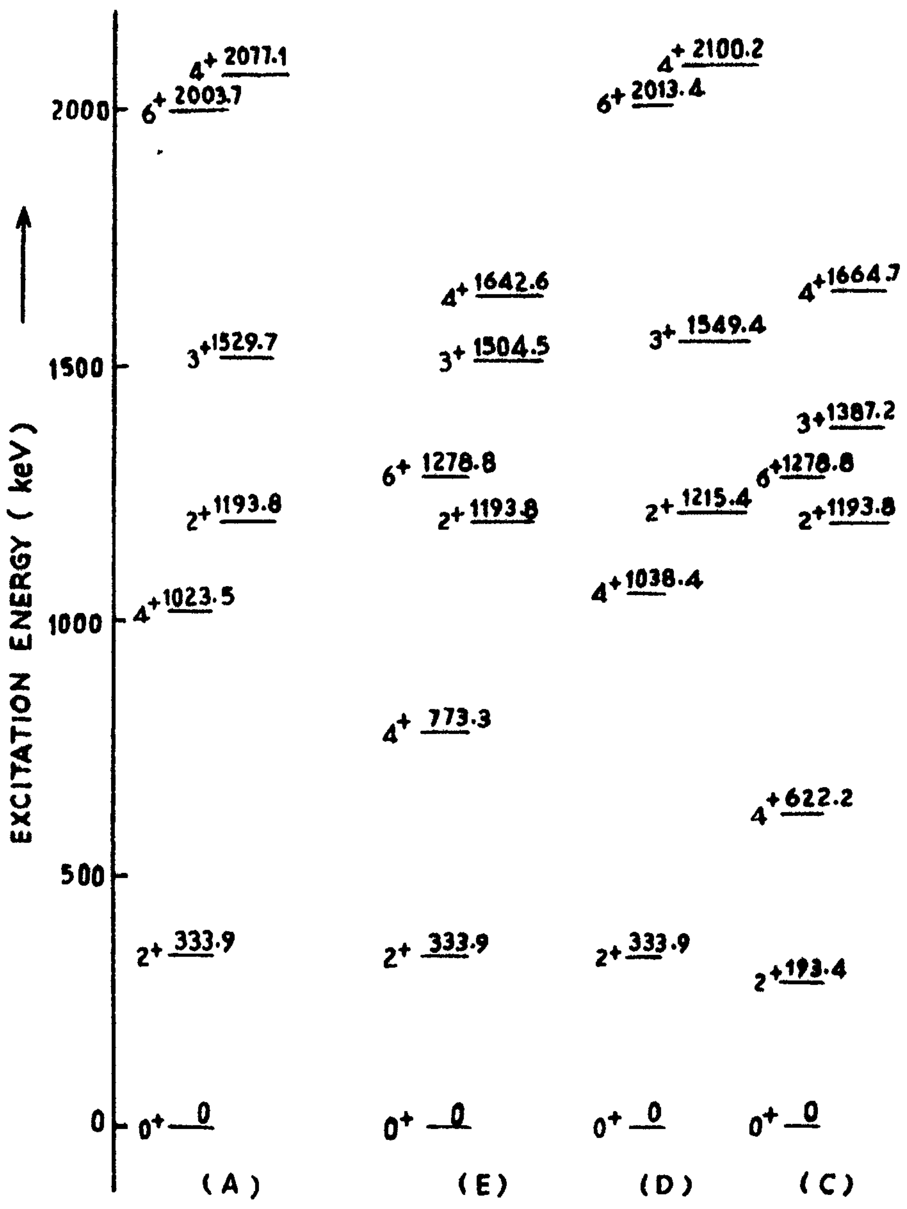

Fig. 4.3 Energy spectrum for ^{150}Sm nucleus.

observed values are in opposite directions for 3^+ and 4^+ states of gamma band while the above equation implies that the deviations should be in the same direction for all energy levels. The effect of β-vibrations has been incorporated in a better way in Davydov-Chaban[20] model. Hence the addition of the cubic term in $\varepsilon(^nI, \gamma)$ to right hand side of equation (4.8) to improve the results does not seem to be worthwhile.

The results of Tamura et al.[3] are also subject to this objection since discrepancies between calculated and observed values are in opposite directions for 3^+ and 4^+ states of ^{154}Sm and ^{150}Sm and 2^+ and 3^+ states of ^{152}Sm belonging to gamma band. Rejecting method (c) due to its anomalous description of energy levels of ^{150}Sm nucleus, method (d), claims to predict lower values of $\gamma = 20.25^o$ which is also not to be subjected for BMVRIC, since $\gamma > 15^o$.

4.3(b) <u>Probability of Electric Transitions</u>:

Tables 4.2, 4.3 and 4.4 describe the experimental B(E2) values and branching ratios under column (E) and calculated ARM values under column (A), (B), (C) and (D) employing γ determined from methods (a), (b), (c) and (d) respectively for $^{150,152,154}Sm$ nuclei. Experimental values under column (E) are taken from Tamura et al.[3] The results under column S_1, S_2 and S_3 are the results of Tamura et al.[3], Kumar[1] and Toyama[21] and are listed for the sake of comparison.

TABLE - 4.2

Individual B(E2) values in units of $(e^2.b^2)$ and branching ratios for ^{150}Sm nucleus.

Transition	Exptl.(E)	S_1	S_2	A	D	C
$2^+ \rightarrow 0^+$	0.274 (6)	0.275	0.233	0.256	0.264 (7)	0.258
$4^+ \rightarrow 2^+$	0.53 (6)	0.51	0.431	0.375	0.388(11)	0.377
$2^{+'} \rightarrow 0^+$	0.0088 (20)	0.020	0.01	0.015	0.0186(5)	0.018
$2^{+'} \rightarrow 2^+$	0.0387 (141)	0.024	0.125	0.051	0.108 (3)	0.112
$2^{+'} \rightarrow 4^+$	0.0194 (100)	0.087	0.034	0.00197	0.017	0.0045
$2^{+'} \rightarrow 2^+/0^+$	4.4 (6)	1.18	12.6	3.33	5.83	6.22
$2^{+'} \rightarrow 2^+/4^+$	2.0 (7)	0.27	3.75	6.66	6.33	7.09
$3^+ \rightarrow 2^+/4^+$	0.29 (6)	1.09	0.52	0.54	0.275	0.258
$3^+ \rightarrow 2^{+'}/2^+$	24(5)	4.34	10.2	16.8	14.3	14.5
$4^{+'} \rightarrow 2^+/4^+$	0.050 (7)	0.69	0.028	0.0023	0.0016	0.034
$4^{+'} \rightarrow 3^+/2^{+'}$	3.7 (13)	0.53	0.58	2.13	1.98	1.95

TABLE - 4.3

Individual B(E2) values in units of $(e^2.b^2)$ and branching ratios for ^{152}Sm nucleus.

Transition	Exptl.(E)	S_1	S_2	S_3	D	B	A
$2^+ \to 0^+$	0.670 (15)	0.673	0.648	-	0.652 (15)	0.630	0.640
$4^+ \to 2^+$	1.017 (14)	0.98	0.993	-	0.947 (22)	0.917	0.926
$2^{+\prime} \to 0^+$	0.0163 (11)	0.050	0.022	-	0.034	0.040	0.029
$2^{+\prime} \to 2^+$	0.0417 (42)	0.053	0.051	-	0.091 (2)	0.381	0.0737
$2^{+\prime} \to 4^+$	0.00416 (32)	0.006	0.003	-	0.0113 (3)	0.042	0.0087
$2^{+\prime} \to 2^+/0^+$	2.44 (13)	1.06	2.33	2.56	2.64	9.48	2.5
$2^{+\prime} \to 2^+/4^+$	11.9 (13)	9.14	19.7	11.1	8.09	8.93	8.47
$3^+ \to 2^+/4^+$	0.95 (7)	2.68	1.42	0.952	0.72	0.181	0.694
$4^{+\prime} \to 2^+/4^+$	0.088 (13)	0.34	0.16	-	0.053	0.0277	0.069

TABLE - 4.4

Individual B(E2) values in units of $(e^2.b^2)$ and branching ratios for ^{154}Sm nucleus

Transition	Exptl.(E)	S_1	S_2	S_3	D	B	A/C
$2^+ \to 0^+$	0.922 (40)	0.881	0.94	-	0.9088(417)	0.882(10)	0.910(42)
$4^+ \to 2^+$	1.186 (39)	1.25	1.40	-	1.304 (40)	1.286(59)	1.306(60)
$2^{+\prime} \to 0^+$	0.013 (13)	0.021	<u>0.033</u>	-	0.026 (1)	0.052 (2)	0.024 (1)
$2^{+\prime} \to 2^+$	0.02	<u>0.047</u>	<u>0.047</u>	-	<u>0.047</u> (2)	<u>0.174</u> (1)	<u>0.044</u> (2)
$2^{+\prime} \to 4^+$	0.0008	<u>10^{-5}</u>	0.01	-	0.0038(2)	0.024 (1)	0.0037(2)
$2^{+\prime} \to 2^+/0^+$	1.56	2.23	1.42	2.04	1.82	<u>3.32</u>	1.81
$2^{+\prime} \to 2^+/4^+$	25.0	<u>3916</u>	<u>8.0</u>	<u>11.1</u>	12.34	<u>7.09</u>	<u>11.2</u>
$3^+ \to 2^+/4^+$	2.5	3.45	1.35	<u>1.19</u>	<u>0.39</u>	<u>0.54</u>	<u><0.39</u>
$4^{+\prime} \to 2^+/4^+$	0.055	<u>0.51</u>	<u>0.32</u>	-	<u>0.17</u>	0.033	<u>0.183</u>

We have imposed Kumar's[22] test (i.e. $0.5 <$ enhancement/hinderence factor $F < 2.0$) on the entries of each column, to see the success of different methods used for γ. Tamura fails to accomodate so many transitions such as $2^{+\prime} \rightarrow 0^+$, $2^{+\prime} \rightarrow 4^+$, $2^{+\prime} \rightarrow 2^+/0^+$, $2^{+\prime} \rightarrow 2^+/4^+$, $3^+ \rightarrow 2^+/4^+$, $3^+ \rightarrow 2^{+\prime}/2^+$, $4^{+\prime} \rightarrow 2^+/4^+$, $4^{+\prime} \rightarrow 3^+/2^+$ for ^{150}Sm, $2^{+\prime} \rightarrow 0^+$, $2^{+\prime} \rightarrow 2^+/0^+$, $3^+ \rightarrow 2^+/4^+$, $4^{+\prime} \rightarrow 2^+/4^+$ for ^{152}Sm and $2^{+\prime} \rightarrow 2^+$, $2^{+\prime} \rightarrow 4^+$, $2^{+\prime} \rightarrow 2^+/4^+$, $4^{+\prime} \rightarrow 2^+/4^+$ for ^{154}Sm nucleus in his sixth order Boson expansion description calculations of collective states. Kumar has been unsuccessful in explaining $2^{+\prime} \rightarrow 2^+$, $2^{+\prime} \rightarrow 2^+/0^+$ for ^{150}Sm, $2^{+\prime} \rightarrow 0^+$, $2^{+\prime} \rightarrow 2^+$, $2^{+\prime} \rightarrow 4^+$, $2^{+\prime} \rightarrow 2^+/4^+$ and $4^{+\prime} \rightarrow 2^+/4^+$ for ^{154}Sm nucleus. Varshni remained silent on ^{150}Sm nucleus and failed almost everywhere in describing $^{152,154}Sm$ nuclei. Meyertervehn did not get through with $2^{+\prime} \rightarrow 2^+$, $2^{+\prime} \rightarrow 4^+$ and $2^{+\prime} \rightarrow 2^+/4^+$ for ^{150}Sm. Davydov-Filippov did not accomodate $2^{+\prime} \rightarrow 4^+$, $2^{+\prime} \rightarrow 2^+/4^+$, $4^{+\prime} \rightarrow 2^+/4^+$ for ^{150}Sm, $3^+ \rightarrow 2^+/4^+$ and $4^{+\prime} \rightarrow 2^+/4^+$ for ^{154}Sm nucleus. The present work faces the least barriers and is hindered only at $4^{+\prime} \rightarrow 2^+/4^+$ ratio for $^{150,154}Sm$ nuclei.

On comparing the calculated values, we find that $2^{+\prime} \rightarrow 2^+/4^+$, $3^+ \rightarrow 2^+/4^+$, $4^{+\prime} \rightarrow 2^+/4^+$ values following method (d) are better than those of Kumar's. The $2^{+\prime} \rightarrow 2^+$, $2^{+\prime} \rightarrow 4^+$, $4^{+\prime} \rightarrow 2^+/4^+$ values are better in column (D) than the values listed in column (A). Toyama described well few ratios for ^{152}Sm nucleus but he was silent on ^{150}Sm. It appears from his work that it is limited to lower values of γ only and predicts almost a constant value for $2^{+\prime} \rightarrow 2^{+}/0^{+}$ ratio for all well deformed nuclei.

This is however important to note that the experimental values of ratio $3^{+} \rightarrow 2^{+}/4^{+}$ for ^{154}Sm nucleus is reported to be 1.4 and 0.95 in references 21 and 23 respectively which is much lower than the value 2.5 reported in reference 3. The former value reduces the hinderence factor from 6 to 3.5/2.5. It is well known that the comparison of the theoretical and experimental electromagnetic properties presents a much more severe test of a theory than does the comparison of the energy levels. When the electromagnetic transitions get weaker, the pertinent experimental data are normally supplied in the form of branching ratios. Therefore, the comparison of the theoretical branching ratios with experimental ones offers a still more stringent test of the theory. It begins to test the validity of the prediction of rather small components of the wave functions. One is puzzled to see Tamura's prediction devaiting by more than a factor of 150 for the branching ratio $2^{+\prime} \rightarrow 2^{+}/4^{+}$ for ^{154}Sm nucleus, while the remarkable success has been achieved throughout in the present work within a factor of 2 only.

The reason for the only discrepancy remained in our work in respect of $4^{+\prime} \rightarrow 2^{+}/4^{+}$ ratio for ^{150}Sm nucleus may be due to neglegience of slight gamma dependence of $E2^{+}$ and $B(E2: 2^{+} \rightarrow 0^{+})$ in DF model in view of uncertainties involved in empirical rule of Grodzins.[17]

$E2^+ \times B(E2: 2^+ \rightarrow 0^+) \approx (2.5 \pm 1) \times 10^{-3} Z^2 A^{-1} [\mathrm{MeV}\ (e^2.b^2)]$

from which equation (4.4) is derived. Again $4^{+\prime} \rightarrow 2^+/4^+$ value changes abruptly at $\gamma = 20^0$ in ARM.

4.3(c) Branching Ratios:

Various theoretical predictions of B(E2) branching ratios viz $2^{+\prime} \rightarrow 0^+/2^+ \rightarrow 0^+$, $2^{+\prime} \rightarrow 2^+/2^+ \rightarrow 0^+$, $2^{+\prime} \rightarrow 4^+/2^+ \rightarrow 0^+$, $2^{+\prime} \rightarrow 2^+/0^+$, $2^{+\prime} \rightarrow 2^+/4^+$, $3^+ \rightarrow 2^+/4^+$ and $4^{+\prime} \rightarrow 2^+/4^+$ are plotted for $^{150,152,154}Sm$ nuclei alongwith their experimental values in Figures 4.4 and 4.5. In these figures E represent experimental[3] results while T and K are calculations of Tamura et al[3] and Kumar[1] results respectively. Our results are shown by curves D. The decreasing trend in experimental values of $2^{+\prime} \rightarrow 0^+/2^+ \rightarrow 0^+$ against neutron number N does not coincide with that of Kumar's and Tamura's values. Tamura's trend is in opposite direction with that of experimental in describing $2^{+\prime} \rightarrow 2^+/0^+$ ratio. For $2^{+\prime} \rightarrow 2^+/4^+$ ratio Kumar's and Tamura's values are again in contradiction with experimental trend. For $3^+ \rightarrow 2^+/4^+$, Tamura's results are slightly better but in $4^{+\prime} \rightarrow 2^+/4^+$ ratio he fails misearably in both quality and quantity. As compared with the other results our results are more closer to the experimental results.

Values of γ calculated using various method are tabulated in table 4.1. Our values are shown in column (D) and are slightly larger than DF values shown in column (A) in ^{152}Sm and ^{154}Sm nuclei. This may be accounted for vibration

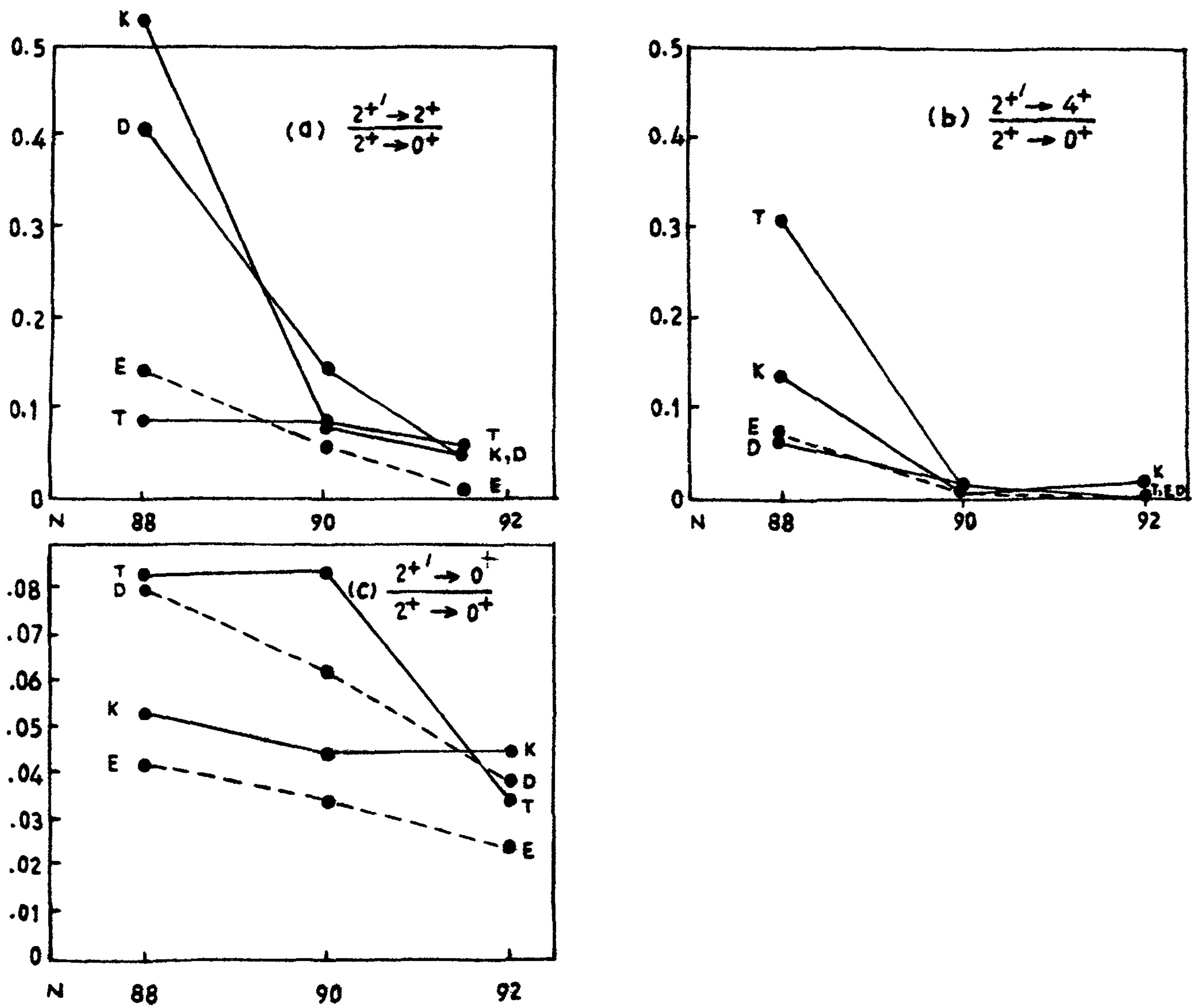

Fig. 4.4 Plot of B(E2) ratios (a) $2^{+'} \to 2^+/2^+ \to 0^+$ (b) $2^{+'} \to 4^+/2^+ \to 0^+$ and (C) $2^{+'} \to 0^+/2^+ \to 0^+$ of Samarium nuclei as a function of neutron number.

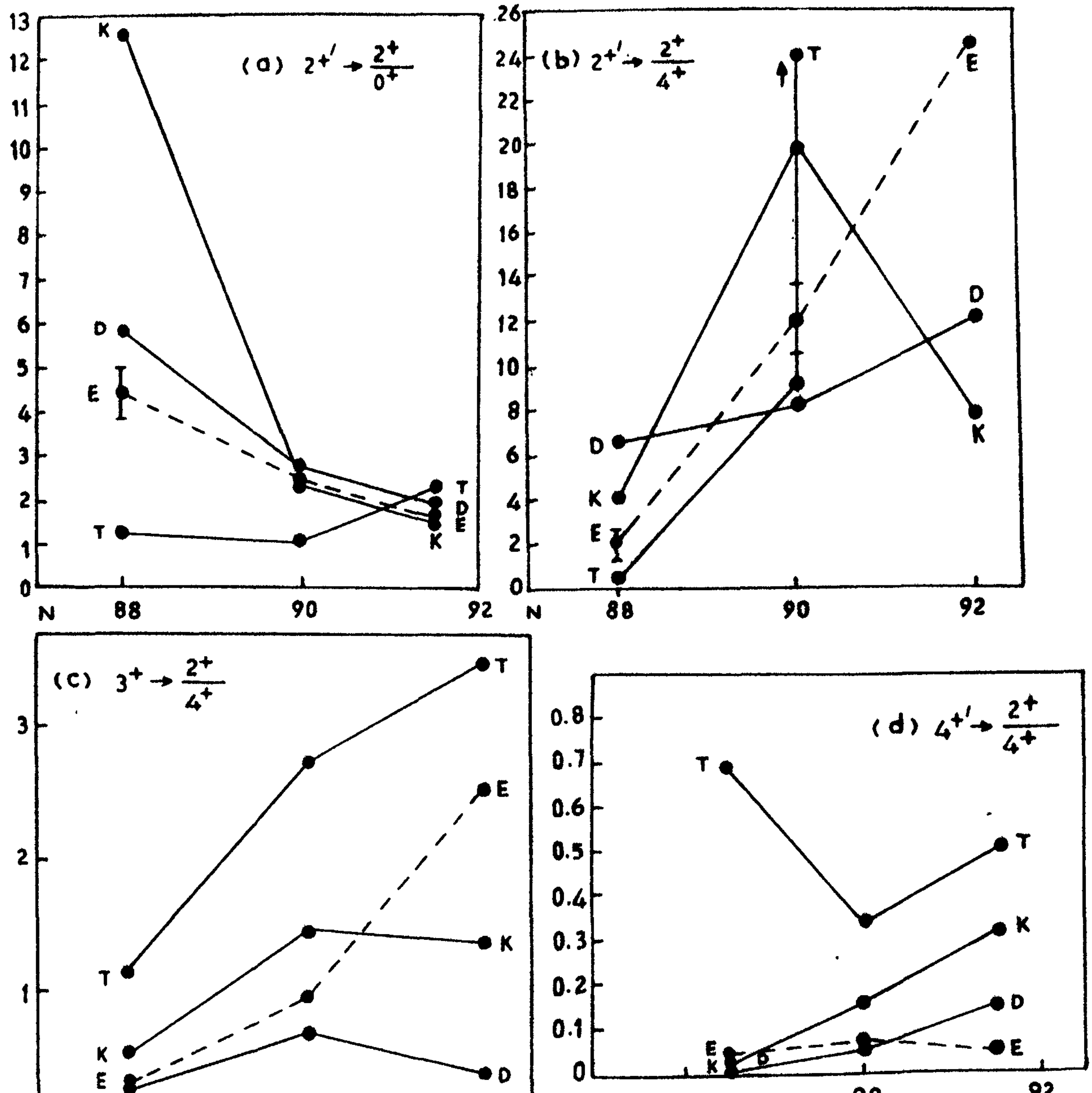

Fig. 4.5 A plot of B(E2) branching ratio (a) $2^{+'} \rightarrow 2^{+}/0^{+}$, (b) $2^{+'} \rightarrow 2^{+}/4^{+}$, (c) $3^{+} \rightarrow 2^{+}/4^{+}$ and (d) $4^{+'} \rightarrow 2^{+}/4^{+}$ of Samarium nuclei as a function of neutron number.

rotation interaction correction and a centrifugal stretching correction. They are of much importance in DF calculations with the analogy of molecular spectra where these corrections have the form $-bI^2(I+1)^2$ in the expression of rotational excitation energies and become more and more important as the equilibrium deformation decreases. Hence γ is subject to correction of 1^o or 2^o if it is calculated from the method (a). In order to have modified γ, Varshni et al. put forward the method (b). The method (b) failed misearably as it excludes transitional nucleus ^{150}Sm to obey DF discipline and gives much higher values of γ for ^{152}Sm. On the other hand Meyertervehn (method c) probably tried to reduce γ for nuclei having $\gamma > 20^o$, but he could do so in the case of ^{150}Sm to such an extent that worsened the agreement between theoretical and experimental B(E2) values and described $2^{+'}$, 3^+ and $4^{+'}$ levels anomalously.

In the case of transitional nucleus ^{150}Sm, the value of γ in column (D) is very close to the DF value (column A), while in ^{148}Sm our value is much lower. The reason for this discrepancy in ^{148}Sm can be discussed in the light of nuclear coupling scheme, which suggests that near closed shell, there is predominently a pairing effect, whereas away from closed shell, simple quadrupole type of interaction predominates. In the transition region both quadrupole and pairing effects contribute, depending largely on the number of particles outside the closed shells.

^{144}Sm (N = 82) is a magic nucleus. The first few nuclear levels display the characteristic of a stiff spherical nucleus which is relatively hard to be excited. ^{152}Sm and ^{154}Sm give low energy levels characteristic of highly deformed nuclei, whose excitations seem collective in nature. Since the transition probability depends primarily on the deformation of the nucleus and the moment of inertia is sensitive to both the deformation and the pairing interaction, probably it is the reason that method (d) could not give even an approximate value of non-axiality parameter γ in the case of ^{148}Sm, which is also called a vibrational nucleus ($|\beta A^{2/3}|$ = 4.07).

Although there is no convincing theoretical reason against non-axially symmetric nuclei in the region with γ about 20^o or so[24)] but even for lesser value of γ , the $2^{+\prime}$ state lies very high with respect to 2^+ state in $^{152,154}Sm$ nuclei (Fig. 4.6) and this too supports the tri-axial nature of the nuclei. Fig. 4.7 shows a linear trend between our γ and Q_o values for $^{150,152,154}Sm$ nuclei. It is interesting to note that in the cases where, nuclei can no longer be considered deformed in the original sense used by Bohr and Mottelson (i.e. when $\gamma > 24^o$), the simple linear relationship ends (Fig. 4.7). In these cases the nuclear coupling scheme would no longer involve a simple one parameter coupling scheme, but would instead involve a

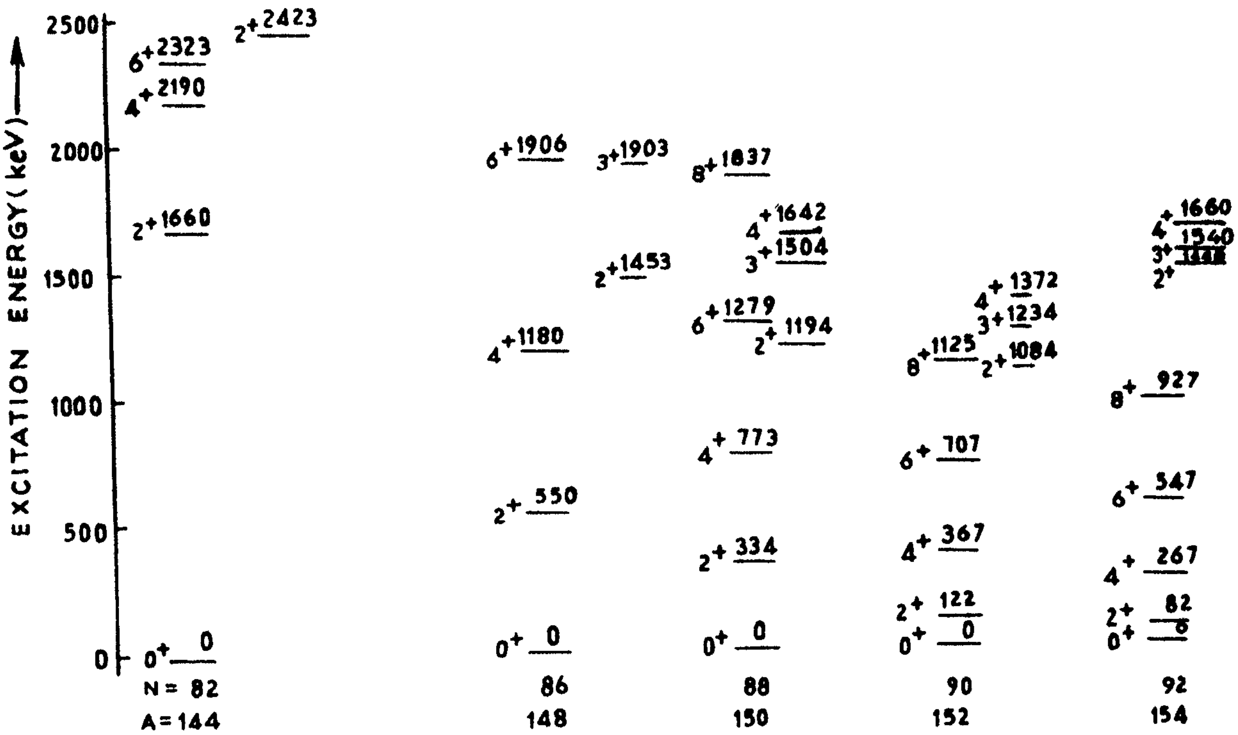

Fig. 4.6 The energy level diagram of the low-lying ground state and gamma vibrational levels of the even isotopes of samarium.

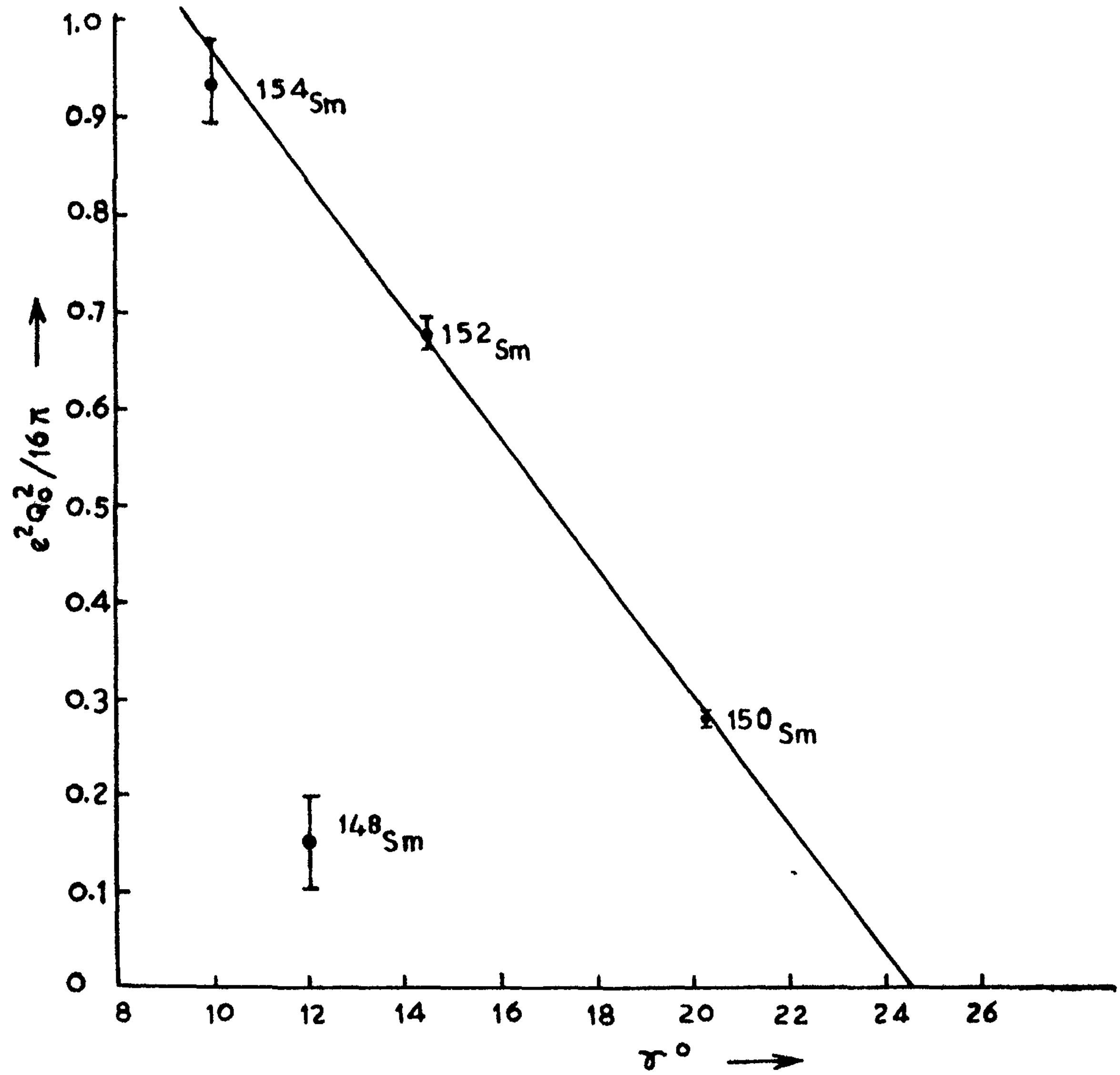

Fig.4.7 Plot of model dependent $e^2Q_0^2/16\pi\,(e^2b^2)$ versus γ.

competition between the quadrupole coupling and the pairing correlation, and the nuclei become pseudo-spherical (i.e. $\beta = 0$).

Upholding the same nuclear coupling scheme for ^{148}Sm nucleus which yields $\gamma = 22.4^o$ at $e^2 Q_o^2/16\pi = 0.147$ ($e^2.b^2$) from the linear relationship (Fig. 4.7), we see that this value of γ is about one degree less than the γ calculated from method (a). The theoretical Q_2 values expressed in (e.b) units for the four isotopes $^{148,150,152,154}Sm$ are -0.556, -0.918, -1.62 and -1.92 which agree with their respective experimental values[25] - 0.73(38), -1.22(22), -1.8 (6) and -2.14 (10) within the experimental errors. This fact supports the view point that ^{148}Sm seems to be classified as transitional rather than its traditional classification as a vibrational nucleus.

Our calculations lend support to the triaxial nature of $^{150,152,154}Sm$ nuclei . In the range $20^o < \gamma < 30^o$ the present work clearly indicates that ^{150}Sm is transitional nucleus and has rather stable tri-axial shape in contradiction to the expectation of very soft fluctuating shape based on the collective potential calculations.

Known B(E2) values, branching ratios and excitation energies in respect of Samarium isotopes have been excellently explained using γ determined from method (d), which are slightly modified than those of usual values of γ (method a).

Rigid rotor model estimates in the units of $(e^2.b^2)$ of various unknown individual transitions and branching ratios in respect of $^{148,150,152,154}Sm$ nuclei are given in Table 4.5. The predicted meanlives are in Table 4.6.

4.4 Conclusion

Extensive calculations of low-lying rotational band and gamma vibrational band energy levels, B(E2) values and branching ratios fro $^{148,150,152,154}Sm$ nuclei employing rigid rotor model with a new technique of determining γ have been done. The non-axiality parameter γ for $^{150,152,154}Sm$ nuclei varies linearly with the model dependent intrinsic quadrupole moment Q_o. This linear relationship rectified the usual values of γ (method a) by enhancing it a few degrees at about $\gamma \approx 15^o$ and reducing a few degrees at $20^o < \gamma < 30^o$ which has been a necessity and stimulated Varshni et al.[10], Meyertervehn[11] to adopt other methods of evaluating γ. The γ thus obtained not only removes the problem of anomalous description of gamma band energy levels but also excellently explains the individual B(E2) values and B(E2) branching ratios within a factor of 2. Analysing Tamura's results in respect of energy spectra of gamma vibrational band for all the Samarium isotopes and the branching ratios where almost everywhere the theory fails misearably, one is forced to conclude that the physical origin of the quadrupole and octupole collective motions

TABLE - 4.5

Rigid Rotor Model estimates in the units of $e^2.b^2$ of various unknown individual Transitions and Branching Ratios in respect of $^{148,150,152,154}Sm$ nuclei.

Transition	Nucleus			
	^{148}Sm	^{150}Sm	^{152}Sm	^{154}Sm
$2^{+\prime} \rightarrow 4^+$	0.0095 (4)			
$2^{+\prime} \rightarrow 2^+/0^+$	10.77			
$2^{+\prime} \rightarrow 4^+/2^+$	0.104			
$3^+ \rightarrow 4^+$	0.0118 (5)	0.115 (3)	0.082 (3)	0.032 (1)
$3^+ \rightarrow 4^+/2^+$	7.76			
$3^+ \rightarrow 2^+/2^{+\prime}$	0.057		0.051	0.0075
$4^{+\prime} \rightarrow 4^+/2^+$	30.3			
$4^{+\prime} \rightarrow 3^+/2^{+\prime}$	1.51		2.165	2.23
$6^+ \rightarrow 4^+$	0.245 (10)	0.461 (13)	1.07 (2)	1.446 (66)
$4^{+\prime} \rightarrow 2^{+\prime}/2^+$	180	1082	65.9	52.3
$3^+ \rightarrow 2^+$	0.0146 (6)	0.033 (1)	0.0596 (14)	0.012
$3^+ \rightarrow 2^{+\prime}$	0.248 (10)	0.473 (13)	1.062 (25)	1.65 (7)
$4^+ \rightarrow 2^{+\prime}$	0.0053 (2)	0.0094 (2)	0.0063 (1)	0.0021
$4^{+\prime} \rightarrow 2^+$	0.0073 (3)	0.0004	0.0057 (1)	0.010

Contd...........

$4^{+\prime} \rightarrow 2^{+\prime}$	0.067 (3)	0.135 (4)	0.375 (1)	0.538(25)
$4^{+\prime} \rightarrow 3^{+}$	0.105 (4)	0.269 (1)	0.81 (2)	1.199(55)
$4^{+\prime} \rightarrow 4^{+}$	0.046 (2)	0.088 (2)	0.107 (2)	0.058 (3)
$6^{+} \rightarrow 4^{+\prime}$	0.0047 (2)	0.0136 (4)	0.022	0.0072
		EMPIRICAL ESTIMATES^{+}		
$8^{+} \rightarrow 6^{+}$	0.228	0.435	1.074	1.497
$10^{+} \rightarrow 8^{+}$	0.2347	0.447	1.103	1.538
$12^{+} \rightarrow 10^{+}$	0.2385	0.455	1.123	1.565
$14^{+} \rightarrow 12^{+}$	0.242	0.460	1.136	1.584
$16^{+} \rightarrow 14^{+}$	0.244	0.464	1.147	1.598
$18^{+} \rightarrow 16^{+}$	0.246	0.468	1.155	1.61

$^{+}$Empirical estimates are obtained using empirical relation[5)] and the modified form of figures drawn in reference 26.

TABLE - 4.6

Predicted meanlives for excited rotational levels of $^{150,152,154}Sm$ nuclei.

Nucleus	I^{π}	Energy (MeV)	Meanlife (p.sec)
^{150}Sm	6^+	1.278	5.20
^{152}Sm	12^+	2.158	1.79
^{154}Sm	10^+	1.338	3.23
	12^+	1.817	2.15

can not be different from one another, as assumed in his theory. At the same time present work indicates that $^{150,152}Sm$, the so called transitional nuclei have tri-axial shapes which are more stable than expected from theoretical potential energy surfaces and thus lends new support to the old Davydov-Filippov model. Quantitative agreement with experimental values indicates that the effect of β- and γ-fluctuations in the range $20^{o} < \gamma < 30^{o}$ which leads to an overall compression of the energy spectrum and affects transition probabilities has been accounted as a modification in the value of γ in the linear behaviour shown in Fig.4.6.

It is inferred that the mass number A and charge Z also play an important role in assigning shape to the nucleus in terms of the asymmetric parameter γ. Our calculations assign 10^{+} and 12^{+} spins to known levels 1.338 MeV and 1.817 MeV in case of ^{154}Sm nucleus and 12^{+} spin for 2.158 MeV observed energy level in ^{152}Sm nucleus.

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CHAPTER - V

PROBABLE MEANLIVES OF ROTATIONAL LEVELS

5.1 Introduction:

The measurement of B(E2) values, meanlives and energies of excited states in nuclei are most active areas of nuclear structure physics. In the last few years it has been a major research point, in developing new experimental techniques, which can provide data giving unique and vital information thus playing a major role in our understanding of nuclear structure. A knowledge of approximate values of meanlives is of great importance to the experimentalists. There has also been a revival of interest in the Asymmetric Rotor Model (ARM) recently[1-13]. The reason for this increase of attention is, partly, the expectation that it may provide a reasonable phenomenological description of a nucleus in some domain of high angular momentum , I.

The striking success of empirical relation[11] compared with the other existing approaches in describing the B(E2) values of gamma-ray cascades has contradicted the axial symmetry in the nucleus at lower spins[2]. The assumption of rigid tri-axial shape with fixed shape parameter β and γ can be considered as an approximation to the actual nuclear wave functions and this has been proved to be very useful and is well supported by new data, most of them obtained from heavy ion experiments during the last decade. The rigid nucleus begins to acquire some freedom at angular momentum I = 6 and consequently obeys Davydov-Rostovsky's[14] (DR) model. This brings $B(E2)_{DR}$ values of gamma-ray cascades close to the systematic trend when plotted against the non-axiality parameter γ. This significant change is useful in determining nuclear shape. These systematics are of immense importance to experimentalists, since by using them one can predict the meanlives of rotational levels, absolute B(E2) values of gamma-ray cascades and beta band head energies. A large number of nuclei in the range $15^\circ < \gamma < 25^\circ$ and belonging to medium mass region have been considered, which reflect the systematics of ARM. It is a remarkable point here that in this region there is no sound theoretical argument against the tri-axial shape of the nucleus.

Earlier work[15-18] is limited to the nuclear region which has small γ values and for small number of transitional nuclei in the rare earth and actinide regions. In those cases the characteristic properties of ARM were not reflected on the systematics. We have used the ARM dependent Q_o, interms of easily observable transition probabilities $B(E2: I_i \rightarrow I_f)$, given by

$$e^2Q_o^{\,2} = 16\pi\,[B(E2:\ 2^+\rightarrow 0^+)+B(E2:\ 2^{+\prime}\rightarrow 0^+)] \qquad [5.1]$$

to keep consistency in the theoretical predictions.

Using this model dependent Q_o we have predicted absolute B(E2) values, meanlives of excited rotational levels upto $12^+\rightarrow 10^+$ transitions and beta band head energies $E0^{+\prime}$.

5.2 Method of Calculation :

Electric quadrupole transition probabilities for transitions inside the ground rotational band between two states of spin I and I' are described by the following empirical formula[11)]

$$B(E2:\ I\rightarrow I')_{emp} = \frac{5e^2Q_o^2}{32\pi}(2I00|I'0)^2\left\{1+\frac{3-2\mathrm{Sin}^2(3\gamma)}{[9-8\mathrm{Sin}^2(3\gamma)]^{1/2}}\right\} \qquad [5.2]$$

where $(2I00|I'0)$ are Clebsch-Gordan Coefficients in the notation $(2Jmm'|J'm')$.

For transitions between two states of ground rotational band, the Clebsch-Gordan Coefficients have the form

$$(2I00|I'0)^2 = \frac{3}{2}\ \frac{(I+1)\ (I+2)}{(2I+3)\ (2I+5)} \qquad [5.3]$$

in decay or de-excitation from $I+2\rightarrow I$.

Hence

$$B(E2:\ I+2\rightarrow I)_{emp} = \frac{5e^2Q_o^2}{32\pi} \times \frac{3}{2}\ \frac{(I+1)\ (I+2)}{(2I+3)(2I+5)} \times$$

$$\left\{1+\frac{3-2\mathrm{Sin}^2(3\gamma)}{[9-8\mathrm{Sin}^2(3\gamma)]^{1/2}}\right\} \qquad [5.4]$$

For $2^+ \rightarrow 0^+$ transition in the rotational band

$$B(E2:\ 2^+ \rightarrow 0^+)_{emp} = \frac{e^2 Q_o^{\ 2}}{32\pi} \left\{ 1 + \frac{3 - 2\mathrm{Sin}^2(3\gamma)}{[9 - 8\mathrm{Sin}^2(3\gamma)]^{1/2}} \right\} \qquad [5.5]$$

$$= B(E2:\ 2^+ \rightarrow 0^+)_{DF} \qquad [5.6]$$

Also

$$B(E2:\ 4^+ \rightarrow 2^+)_{emp} = \frac{10}{7} \frac{e^2 Q_o^{\ 2}}{32\pi} \left\{ 1 + \frac{3 - 2\mathrm{Sin}^2(3\gamma)}{[9 - 8\mathrm{Sin}^2(3\gamma)]^{1/2}} \right\} \qquad [5.7]$$

$$= \frac{10}{7} B(E2:\ 2^+ \rightarrow 0^+)_{DF}$$

Similar expressions for empirical transition probabilities can be derived from equation (5.4) for $6^+ \rightarrow 4^+$, $8^+ \rightarrow 6^+$, $10^+ \rightarrow 8^+$ and $12^+ \rightarrow 10^+$ transitions inside the ground rotational band.

$B(E2)_{emp}$ values for the transitions considered in the present work have been calculated by the above equations, using model dependent intrinsic quadrupole moment Q_o.

$B(E2)_{exp}$ values have been calculated from the meanlives using the relation[19]

$$B(E2:\ I{+}2 \rightarrow I)_{exp} = \frac{0.0812}{E_\gamma^5 \ (1 + \alpha_T)\ \tau} \qquad [5.8]$$

where E_γ, the excitation energy in MeV, τ, the meanlife of the excited state in peco seconds and α_T, the total internal conversion coefficients have been calculated from

Table of Isotopes[20].

$B(E2)_{DR}$ values for these transitions have been calculated by using the relation[14)]

$$B(E2:I\to I')_{DR} = \frac{5e^2Q_o^2}{16\pi}(2I00|I'0)^2(1-\frac{1}{s})\ (1-\frac{2s}{3q^2})^2 \qquad [5.9]$$

where $\quad s = \frac{E2^{+\prime}}{E2^{+}}$ and $q = \frac{E0^{+\prime}}{E2^{+}}$

$E2^+$, $E2^{+\prime}$ and $E0^{+\prime}$ are the energies of first 2^+ excited state, second 2^+ excited state and first 0^+ excited states respectively and have been taken from Table of Sakai and Rester[21].

5.2(a) <u>Probable B(E2) Values and Meanlives</u>:

Table 5.1 illustrates the comparison of $B(E2)_{emp}$, $B(E2)_{exp}$ and $B(E2)_{DR}$ values for a number of even-even nuclei in $4^+\to 2^+$, $6^+\to 4^+$, $8^+\to 6^+$, $10^+\to 8^+$ and $12^+\to 10^+$ transitions. Using empirical relations as discussed in section 5.2 and model dependent Q_o, $B(E2)_{emp}$ values were estimated for a large number of cases. The $B(E2)_{exp}$ values were also estimated using experimental values of meanlives, E_γ and α_T. In figures 5.1 to 5.6, we have plotted the ratio $F_{emp} = B(E2)_{exp}/B(E2)_{emp}$ versus γ, which was calculated by most widely used method[22]. In the same figures we have also plotted the ratio $F_{DR} = B(E2)_{exp}/B(E2)_{DR}$. Equation (5.9)is used to calculated the $B(E2)_{DR}$ values for the transitions considered.

TABLE - 5.1

Comparison of $B(E2)_{emp}$, $B(E2)_{exp}$ and $B(E2)_{DR}$ values for Gamma-Ray Cascades

Nucleus	Transition	Gamma-Ray Energy (E_γ) (MeV)	Meanlife τ (p.sec.)	$B(E2)_{exp}$ ($e^2.b^2$)	$B(E2)_{emp}$ ($e^2.b^2$)	$B(E2)_{DR}$ ($e^2.b^2$)	$\frac{e^2Q_o^2}{16\pi}$	γ (degree)
^{102}Mo	$4^+ \to 2^+$	0.447	18.04 (38)	0.253 (42)	0.296	0.057	0.218	18.5
^{98}Ru	$4^+ \to 2^+$	0.7454	3.27 (36)	0.108 (12)[a]	0.128	0.053	0.0815	27.0
^{100}Ru	$4^+ \to 2^+$	0.6869	366 (36)	0.146 (15)	0.159	0.034	0.1077	24.5
^{102}Ru	$4^+ \to 2^+$	0.6313	40.4 (72)	0.201 (38)	0.200	0.013	0.1333	25.5
^{104}Ru	$4^+ \to 2^+$	0.5305	8.66 (60)	0.217 (4)	0.250	0.086	0.1659	24.5
^{102}Pd	$4^+ \to 2^+$	0.7195	2.86 (17)	0.147 (9)[a]	0.137	0.067	0.095	23.0
^{104}Pd	$4^+ \to 2^+$	0.7678	2.00 (12)	0.152 (9)[a]	0.161	0.065	0.1082	25.0
^{106}Pd	$4^+ \to 2^+$	0.7174	2.02 (29)	0.22 (3)	0.197	0.047	0.1276	26.5
^{108}Pd	$4^+ \to 2^+$	0.6142	3.32 (43)	0.28 (4)	0.235	0.063	0.1494	27.0
^{110}Pd	$4^+ \to 2^+$	0.547	5.5 (9)	0.31 (4)	0.275	0.079	0.1746	27.0

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^{108}Cd	$4^+ \rightarrow 2^+$	0.8755	1.33 (19)	0.120 (15)	0.127	0.038	0.0868	24.5
^{112}Cd	$4^+ \rightarrow 2^+$	0.7968	1.29 (14)	0.194 (19)	0.173	0.033	0.1098	27.0
^{114}Cd	$4^+ \rightarrow 2^+$	0.725	1.92 (32)	0.212 (36)[b]	0.186	0.166	0.1184	27.0
^{116}Cd	$4^+ \rightarrow 2^+$	0.7056	2.45 (57)	0.191 (36)	0.184	0.055	0.1224	25.5
^{122}Te	$4^+ \rightarrow 2^+$	0.6165	5.09	0.179[b]	0.207	0.079	0.1359	26.2
^{120}Xe	$4^+ \rightarrow 2^+$	0.472	8.8 (19)	0.85 (82)	0.275	-	0.189	23.5
^{122}Xe	$4^+ \rightarrow 2^+$	0.4974	8.2 (13)	0.325 (51)	0.335	-	0.229	24.0
	$6^+ \rightarrow 4^+$	0.6389	3.9 (7)	0.19 (3)	0.410	-		
^{134}Ba	$4^+ \rightarrow 2^+$	0.7957	12.5 (26)	0.0204	0.237	0.094	0.1413	30.0
^{132}Ce	$4^+ \rightarrow 2^+$	0.5222	4.35 (144)	0.437 (109)	0.485	-	0.329	24.5
	$6^+ \rightarrow 4^+$	0.6824	1.15 (86)	0.476 (35)	0.594	-		
^{134}Ce	$4^+ \rightarrow 2^+$	0.6394	4.47 (86)	0.170(24)	0.322	-	0.214	25.5
	$6^+ \rightarrow 4^+$	0.8144	3.0 (13)	0.075 (32)	0.403	-		
	$10^+ \rightarrow 8^+$	0.9178	8.6 (14)	0.015 (2)	0.36	-		
^{140}Ce	$4^+ \rightarrow 2^+$	0.4875	4978	0.0006	0.0604	-	0.036	30.0
	$6^+ \rightarrow 4^+$	0.0025	10×10^6 (3×10^6)	-	0.076	-		
^{150}Nd	$4^+ \rightarrow 2^+$	0.2524	88 (7)	0.901 (71)	0.911	0.511	0.67	14.0
	$6^+ \rightarrow 4^+$	0.3387	17.3 (13)	1.00 (7)	0.983	0.56		

Contd.....97....

	$8^+ \rightarrow 6^+$	0.4095	6.8 (6)	1.02 (9)	1.17	0.593		
^{148}Sm	$4^+ \rightarrow 2^+$	0.6299	3.32 (86)	0.248 (64)	0.215	0.041	0.1471	23.7
^{150}Sm	$4^+ \rightarrow 2^+$	0.4394	9.52 (72)	0.521 (39)	0.391	0.070	0.2828	20.5
^{152}Sm	$4^+ \rightarrow 2^+$	0.2448	82.3 (11)	1.03 (1)	0.938	0.61	0.6863	13.0
	$6^+ \rightarrow 4^+$	0.3408	14.7 (4)	1.16 (2)	1.00	0.67		
	$8^+ \rightarrow 6^+$	0.4182	4.4 (2)	1.41 (7)	1.19	0.708		
	$10^+ \rightarrow 8^+$	0.4826	1.99 (18)	1.54 (14)	1.78	0.728		
^{154}Sm	$4^+ \rightarrow 2^+$	0.1848	249 (7)	1.18 (3)	1.30	1.09	0.935	9.5
	$6^+ \rightarrow 4^+$	0.2799	33.6 (10)	1.31 (4)	1.30	1.19		
	$8^+ \rightarrow 6^+$	0.3562	8.9 (8)	1.54 (15)	1.56	1.252		
^{154}Gd	$4^+ \rightarrow 2^+$	0.2481	66.2 (22)	1.18 (4)	1.086	0.743	0.799	14.0
	$6^+ \rightarrow 4^+$	0.3467	11.4 (4)	1.36 (5)	1.17	0.82		
	$8^+ \rightarrow 6^+$	0.4268	3.7 (1)	1.50 (5)	1.40	0.856		
^{156}Gd	$4^+ \rightarrow 2^+$	0.1992	164 (3)	1.29 (2)	1.32	1.06	0.9496	11.0
	$6^+ \rightarrow 4^+$	0.2965	22.8 (6)	1.47 (3)	1.36	1.16		
	$8^+ \rightarrow 6^+$	0.3805	6.2 (4)	1.61 (11)	1.65	1.22		
^{158}Gd	$4^+ \rightarrow 2^+$	0.1819	233 (18)	1.37 (11)	1.361	1.21	0.978	10.5
	$8^+ \rightarrow 6^+$	0.3654	7.36 (57)	1.69 (13)	1.66	1.40		

	$10^+ \rightarrow 8^+$	0.4461	2.60 (14)	1.77 (9)	1.73	1.438		
	$12^+ \rightarrow 10^+$	0.5162	1.41 (11)	1.57 (12)	1.50	1.464		
^{158}Dy	$4^+ \rightarrow 2^+$	0.2184	108 (13)	1.29 (16)	1.385	1.10	1.013	13.0
^{160}Dy	$4^+ \rightarrow 2^+$	0.1970	146 (13)	1.49 (13)	1.40	1.21	1.019	12.0
	$6^+ \rightarrow 4^+$	0.2974	28.4 (14)	1.23 (6)	1.46	1.33		
	$8^+ \rightarrow 6^+$	0.3860	4.9 (4)	1.57 (10)	1.74	1.395		
	$10^+ \rightarrow 8^+$	0.4614	2.3 (2)	1.65 (15)	1.77	1.433		
	$12^+ \rightarrow 10^+$	0.5231	1.6 (2)	1.29 (79)	1.6	1.458		
^{162}Dy	$4^+ \rightarrow 2^+$	0.1850	190 (11)	1.53 (9)	1.52	1.31	1.109	12.0
	$6^+ \rightarrow 4^+$	0.2828	29.5 (34)	1.38 (16)	1.59	1.44		
	$8^+ \rightarrow 6^+$	0.3727	5.90 (50)	1.91 (8)	1.89	1.51		
	$10^+ \rightarrow 8^+$	0.4538	2.3 (2)	1.79 (11)	1.93	1.55		
	$12^+ \rightarrow 10^+$	0.5259	1.3 (7)	1.48 (8)	1.47	1.58		
^{164}Dy	$4^+ \rightarrow 2^+$	0.1689	245	1.57	1.622	-	1.1802	12.0
	$6^+ \rightarrow 4^+$	0.259	43.3 (15)	1.67 (6)	1.697	1.28		
	$8^+ \rightarrow 6^+$	0.3424	9.8 (2)	1.69 (12)	2.02	1.34		
	$10^+ \rightarrow 8^+$	0.4176	3.4 (1)	1.89 (10)	2.05	1.377		
	$12^+ \rightarrow 10^+$	0.484	1.6 (1)	1.87 (3)	1.86	1.402		

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^{156}Er	$4^+ \rightarrow 2^+$	0.4528	7.8 (9)	0.54 (7)	0.48	0.17	0.33	23.5
	$6^+ \rightarrow 4^+$	0.5434	1.6 (9)	1.02 (89)	1.584	0.19		
^{158}Er	$4^+ \rightarrow 2^+$	0.3351	20.8 (10)	0.88 (4)	0.76	0.42	0.56	18.5
	$6^+ \rightarrow 4^+$	0.2631	4.0 (7)	1.15 (72)	0.882	0.46		
	$8^+ \rightarrow 6^+$	0.5229	1.6 (5)	1.28 (45)	1.04	0.484		
	$10^+ \rightarrow 8^+$	0.5788	1.1 (5)	1.13 (66)	0.93	0.497		
	$12^+ \rightarrow 10^+$	0.608	<1.0	0.96	0.80	0.506		
^{160}Er	$4^+ \rightarrow 2^+$	0.2639	49.8 (24)	1.17 (6)	1.14	0.85	0.84	15.0
	$6^+ \rightarrow 4^+$	0.3753	7.8 (7)	1.36 (12)	1.25	0.93		
	$8^+ \rightarrow 6^+$	0.4637	3.2 (7)	1.17 (28)	1.49	0.98		
	$10^+ \rightarrow 8^+$	0.5321	1.7 (7)	1.09 (55)	1.43	1.007		
^{164}Er	$4^+ \rightarrow 2^+$	0.2081	124 (11)	1.40 (13)	1.62	1.37	1.186	13.0
	$8^+ \rightarrow 6^+$	0.4102	3.82 (27)	1.83 (13)	2.06	1.58		
	$10^+ \rightarrow 8^+$	0.4935	1.41 (9)	2.97 (12)	2.05	1.62		
	$12^+ \rightarrow 10^+$	0.5646	1.18 (9)	1.20 (9)	1.84	1.653		
^{166}Er	$4^+ \rightarrow 2^+$	0.1844	168.8 (101)	1.69 (11)	1.57	1.38	1.148	12.5
	$6^+ \rightarrow 4^+$	0.2804	29.6	1.60	1.66	1.52		
	$8^+ \rightarrow 6^+$	0.3658	6.8 (4)	1.85 (11)	1.98	1.593		

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	$10^+ \rightarrow 8^+$	0.4385	2.3 (2)	2.1 (2)	1.99	1.636		
	$12^+ \rightarrow 10^+$	0.4975	1.3 (1)	1.97 (15)	1.79	1.665		
^{168}Er	$4^+ \rightarrow 2^+$	0.1843	173 (12)	1.67 (10)	1.64	1.43	1.195	12.0
	$8^+ \rightarrow 6^+$	0.3795	4.91 (43)	2.10 (18)	2.04	1.647		
^{170}Er	$4^+ \rightarrow 2^+$	0.1815	204 (5)	1.55 (4)	1.66	1.37	1.203	11.5
	$8^+ \rightarrow 6^+$	0.3719	5.16 (37)	2.21 (16)	2.06	1.581		
	$10^+ \rightarrow 8^+$	0.461	2.13 (14)	1.83 (12)	2.11	1.624		
	$12^+ \rightarrow 10^+$	0.542	0.82 (4)	2.12 (10)	1.92	1.653		
^{162}Yb	$4^+ \rightarrow 2^+$	0.3203	20.2 (28)	1.19 (17)	1.01	0.581	0.746	17.0
	$6^+ \rightarrow 4^+$	0.4376	4.6 (9)	1.10 (33)	1.148	0.64		
	$8^+ \rightarrow 6^+$	0.5214	2.02 (7)	1.04 (4)	1.36	0.67		
^{164}Yb	$4^+ \rightarrow 2^+$	0.2624	42.8 (16)	1.37 (5)	1.264	0.97	0.93	15.0
	$6^+ \rightarrow 4^+$	0.3747	7.2 (3)	1.47 (5)	1.39	1.06		
	$8^+ \rightarrow 6^+$	0.463	2.2 (7)	1.73 (65)	1.65	1.12		
	$10^+ \rightarrow 8^+$	0.5306	1.2 (4)	1.61 (62)	1.58	1.15		
	$12^+ \rightarrow 10^+$	0.5767	0.79 (30)	1.59 (71)	1.39	1.171		
^{166}Yb	$4^+ \rightarrow 2^+$	0.2281	76.3 (24)	1.47 (5)	1.45	1.16	1.06	13.0
	$6^+ \rightarrow 4^+$	0.3375	11.2 (4)	1.57 (6)	1.55	1.27		

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	$8^+ \rightarrow 6^+$	0.4303	3.1 (3)	1.75 (2)	1.84	1.334		
	$10^+ \rightarrow 8^+$	0.5107	1.4 (7)	1.65 (109)	1.84	1.370		
	$12^+ \rightarrow 10^+$	0.5697	0.92 (47)	1.46 (102)	1.64	1.395		
^{170}Yb	$8^+ \rightarrow 6^+$	0.3899	4.28 (35)	1.61 (14)	1.79	1.416	1.04	10.8
	$10^+ \rightarrow 8^+$	0.4741	1.67 (11)	2.03 (13)	1.84	1.455		
	$12^+ \rightarrow 10^+$	0.5459	1.11 (9)	1.51 (12)	1.68	1.481		
^{166}Hf	$4^+ \rightarrow 2^+$	0.311	24.2 (16)	1.10 (7)	1.165	0.617	0.86	(16.7)
	$6^+ \rightarrow 4^+$	0.4273	5.0 (7)	1.14	1.31	0.680		
	$8^+ \rightarrow 6^+$	0.509	1.7 (7)	1.38 (56)	1.568	0.711		
	$10^+ \rightarrow 8^+$	0.564	1.0 (7)	1.42 (99)	1.44	0.731		
	$12^+ \rightarrow 10^+$	0.594	1.3 (10)	0.84 (64)	1.26	0.744		
^{168}Hf	$4^+ \rightarrow 2^+$	0.2616	51.9 (58)	1.14 (13)	1.138	0.781	0.836	(14.3)
	$6^+ \rightarrow 4^+$	0.3712	8.5 (9)	1.28 (13)	1.24	0.86		
	$8^+ \rightarrow 6^+$	0.458	2.9 (3)	1.42 (15)	1.47	0.901		
	$10^+ \rightarrow 8^+$	0.522	1.4 (2)	1.50 (21)	1.42	0.926		
	$12^+ \rightarrow 10^+$	0.57	0.75 (2)	1.59 (4)	1.27	0.942		
^{180}Hf	$4^+ \rightarrow 2^+$	0.2153	115.4 (72)	1.32 (17)	1.355	0.788	0.974	10.7
^{182}W	$4^+ \rightarrow 2^+$	0.2293	92.3 (72)	1.235 (90)	1.714	1.21	1.238	11.5

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	$6^+ \to 4^+$	0.3511	12.5 (13)	1.22 (13)	1.77	1.54		
^{184}W	$4^+ \to 2^+$	0.2528	67.8 (58)	0.954 (9)	0.946	0.77	0.695	14.0
	$6^+ \to 4^+$	0.3843	7.8 (8)	1.24 (13)	1.02	1.26		
^{186}W	$4^+ \to 2^+$	0.2742	51.9	0.892	0.945	0.66	0.697	16.0
	$6^+ \to 4^+$	0.412	5.0	1.36	1.05	1.10		
^{184}Os	$4^+ \to 2^+$	0.264	66.4 (187)	0.84 (26)	0.774	0.62	0.569	14.0
^{186}Os	$4^+ \to 2^+$	0.2969	34.6	0.90	0.904	0.65	0.667	16.5
	$6^+ \to 4^+$	0.4348	4.56	1.16	1.02	0.71		
^{188}Os	$4^+ \to 2^+$	0.323	29.2	0.79	0.841	0.53	0.617	19.0
^{190}Os	$4^+ \to 2^+$	0.3611	20.2	0.62	0.774	0.39	0.543	22.0
	$6^+ \to 4^+$	0.5025	2.54	1.03	0.94	0.42		
^{192}Os	$4^+ \to 2^+$	0.3744	20.2 (22)	0.54 (6)	0.69	-	0.419	25.5
	$6^+ \to 4^+$	0.5084	2.74 (43)	0.87 (14)	0.85	0.33		
^{228}Th	$4^+ \to 2^+$	0.1291	232.3 (72)	1.72 (7)	1.96	-	1.41	9.8
^{230}Th	$4^+ \to 2^+$	0.1215	239.5 (72)	2.29 (7)	2.26	1.84	1. 625	10.5

a→ Values taken from reference [Weeks K and Tamura T, Phys. Rev. C22 888 (1980)].

b→ Values taken from reference [Kishimoto T and Tamura T, Nucl. Phys. A270 317 (1976)].

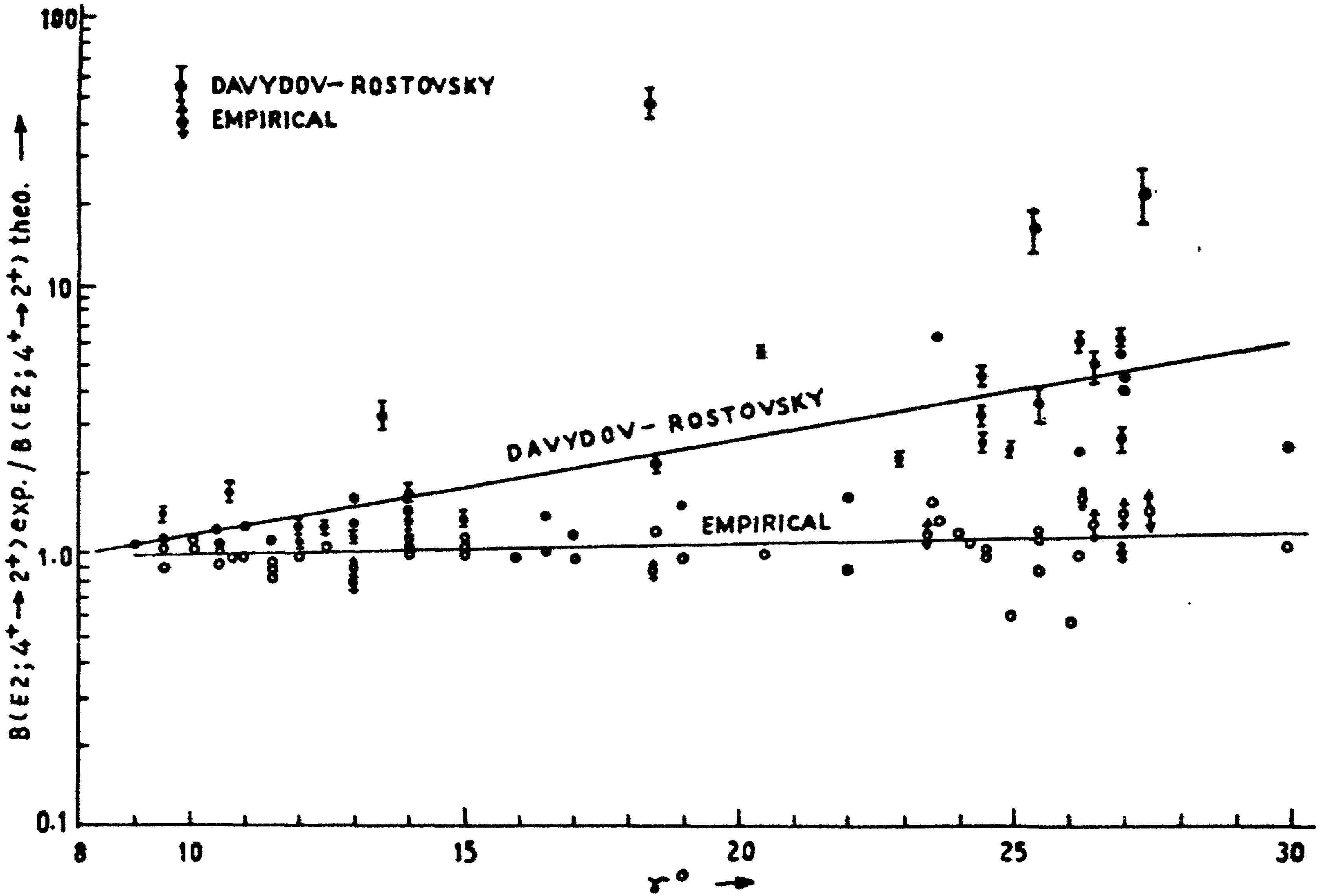

Fig. 5.1 Plot of $B(E2;4^+ \to 2^+)$ exp. / $B(E2;4^+ \to 2^+)$ theo. versus Non-axiality parameter γ.

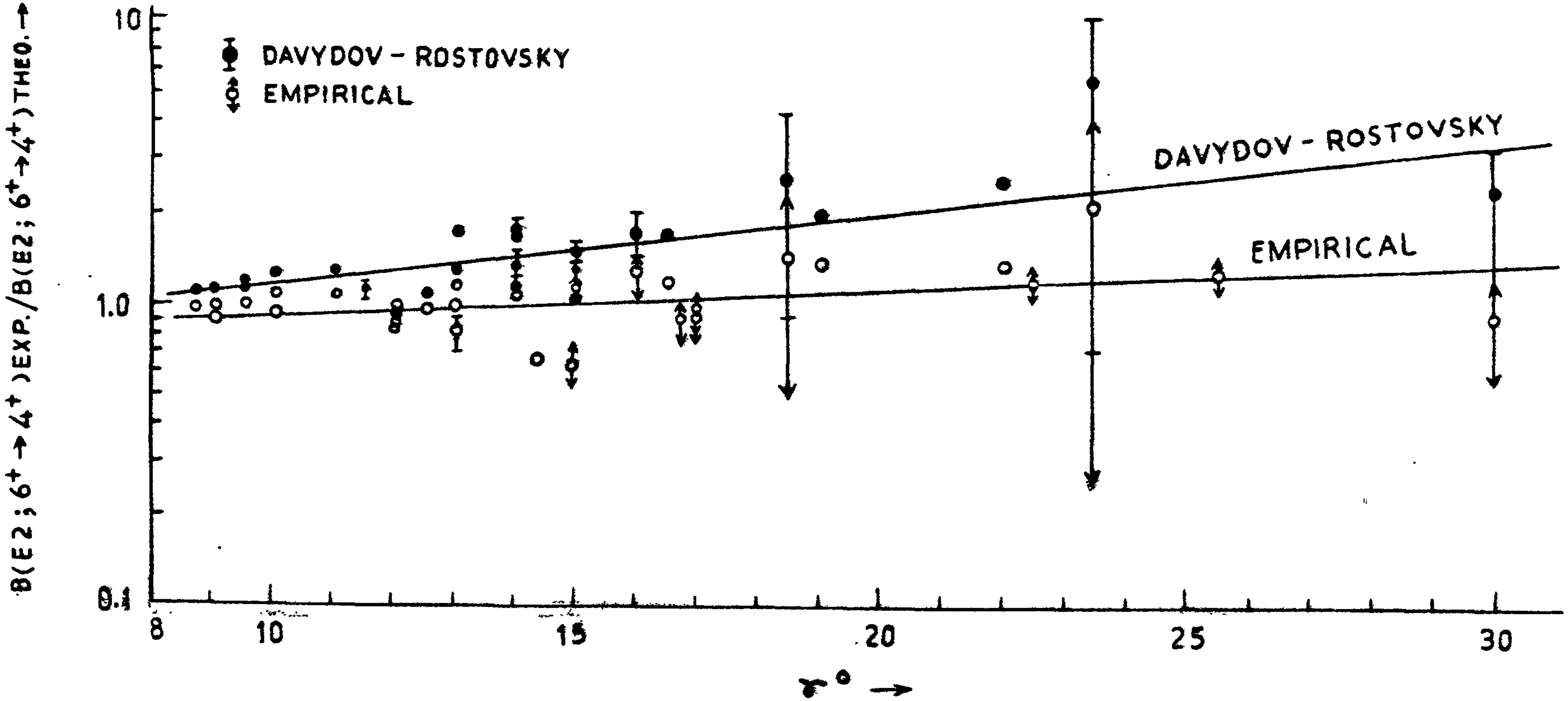

Fig. 5.2 Plot of $B(E2; 6^+ \rightarrow 4^+)$ Exp./ $B(E2; 6^+ \rightarrow 4^+)$ Theo. values against non-axiality parameter γ.

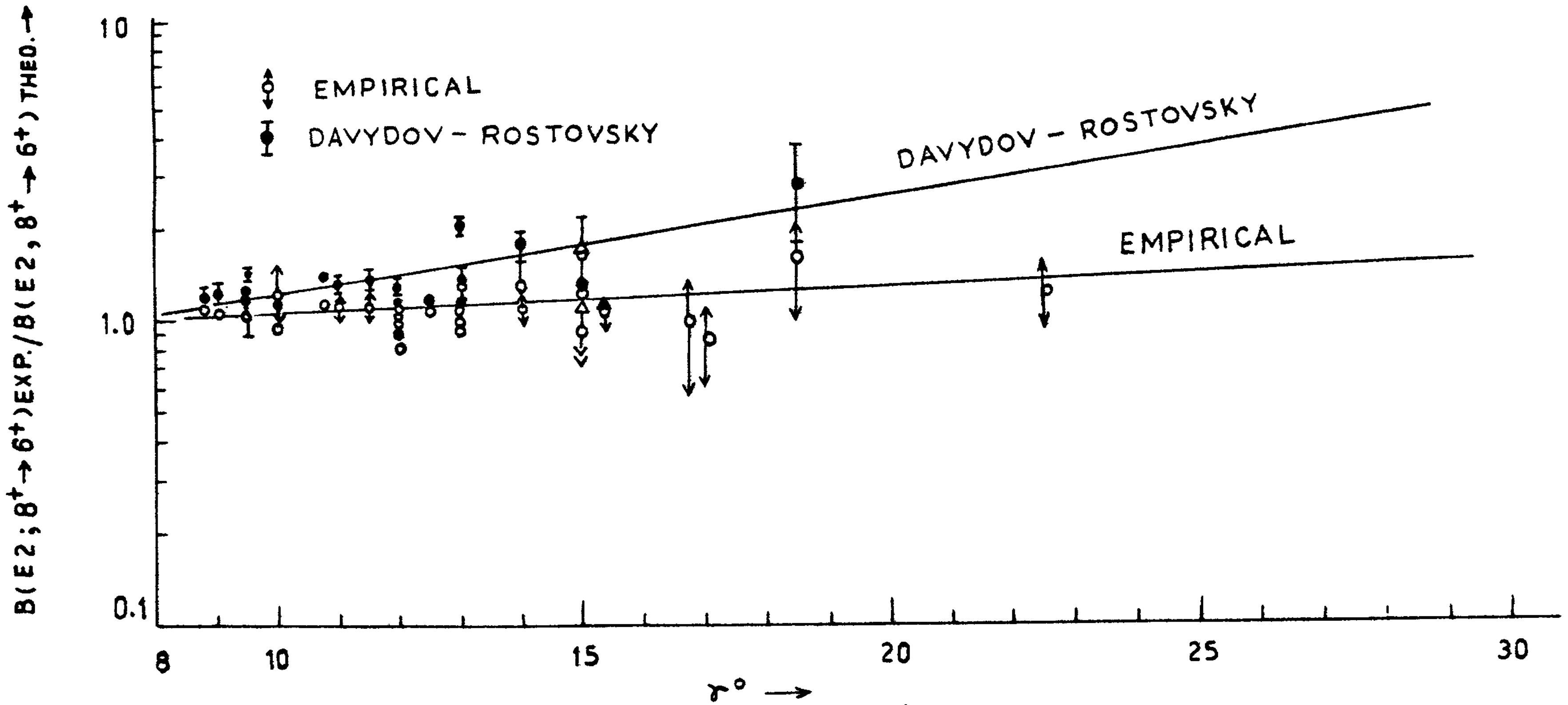

Fig. 5.3 Plot of $B(E2; 8^+ \rightarrow 6^+)$ exp./ $B(E2; 8^+ \rightarrow 6^+)$ theo. values against non-axiality parameter γ.

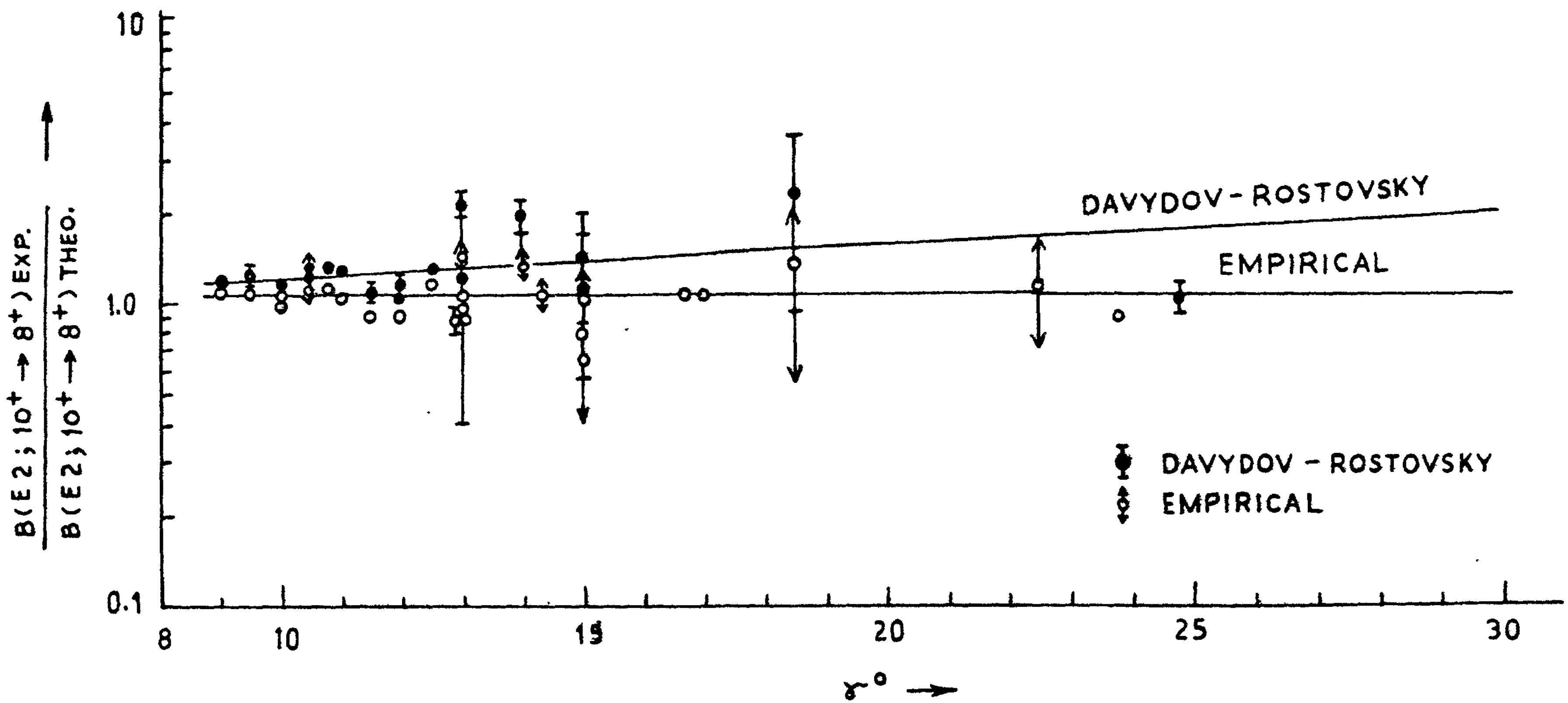

Fig. 5.4 Plot of $B(E2; 10^+ \rightarrow 8^+)$ Exp. / $B(E2; 10^+ \rightarrow 8^+)$ Theo. values against non-axiality parameter γ.

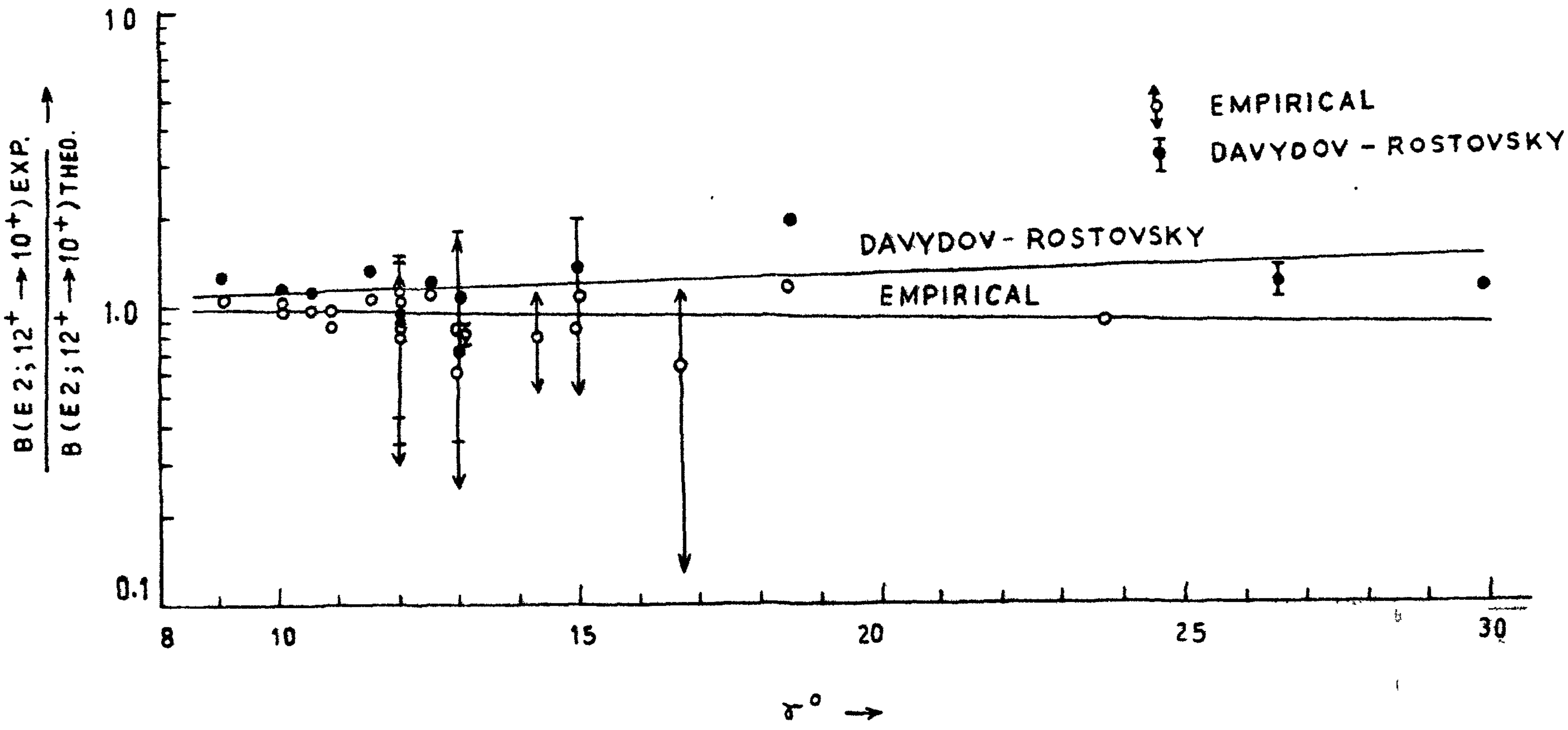

Fig. 5.5 Plot of $B(E2; 12^+ \rightarrow 10^+)$ Exp. / $B(E2; 12^+ \rightarrow 10^+)$ Theo. values against non-axiality parameter γ.

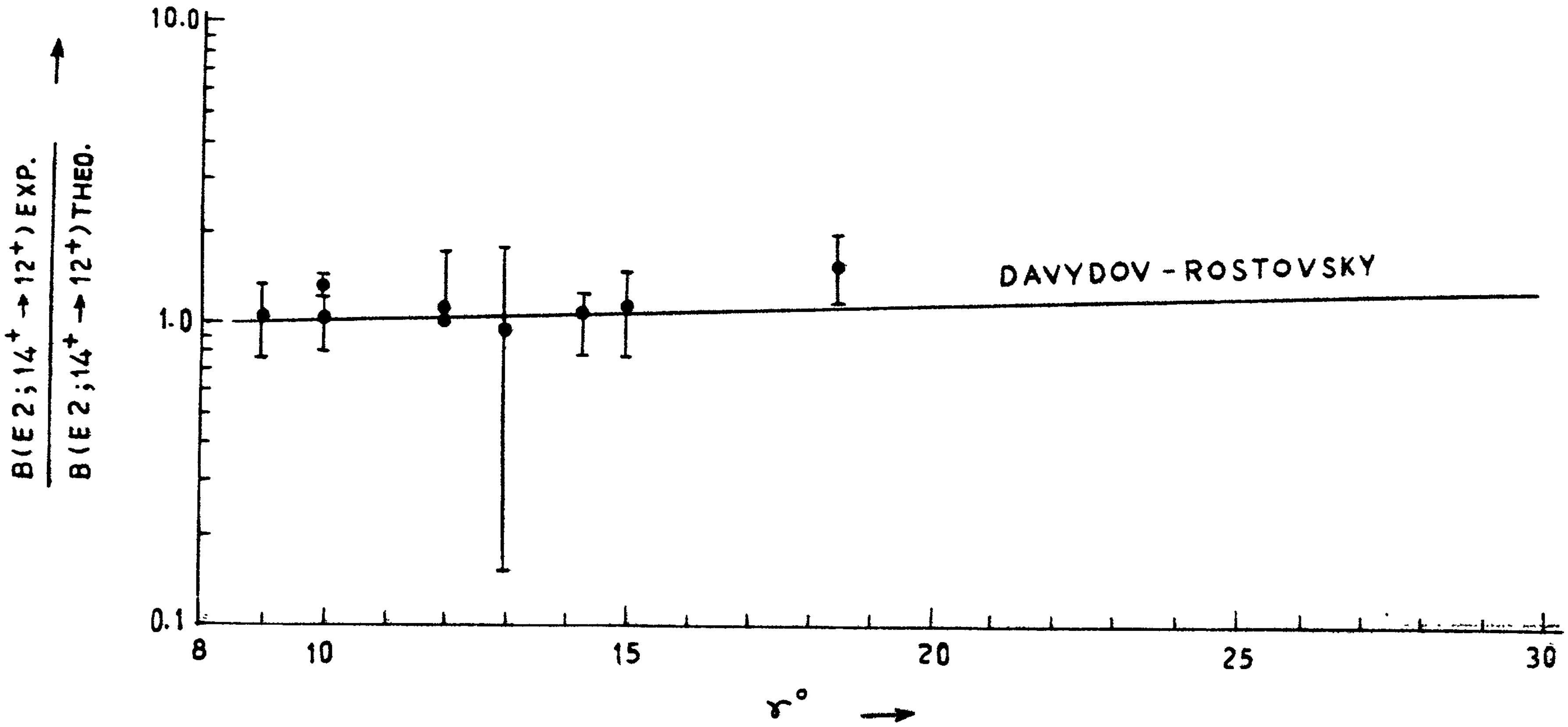

Fig. 5.6 Plot of $B(E2; 14^+ \rightarrow 12^+)$ Exp./$B(E2; 14^+ \rightarrow 12^+)$Theo. values against the non-axiality parameter γ.

It is clear from these figures that the ratio F_{emp} is very close to unity and thus empirical B(E2) values may be used for the prediction of meanlives for those transitions where neither the experimental B(E2) values nor the meanlives are known. These predicted values of meanlives are displayed in Table 5.2. In Tables 5.1 and 5.2 there are some cases for which $E2^{+'}$ level is not experimentally known, the γ values are given in brackets. To determine γ values for such cases a systematic between $AE2^{+}$ and γ was plotted (Fig. 5.7) for a large number of nuclei for which γ can be calculated using known values of $E2^{+}$ and $E2^{+'}$. Using this systematic[23] (Fig.5.7) approximate values of γ were calculated for those nuclei in which $E2^{+'}$ are not known but $E2^{+}$ are known.

5.2(b) Beta Band Head Energy:

A remarkable systematic trend observed in F_{DR} using Davydov-Rostovsky model[14] calculations for $I > 4$ (Fig.5.2 to Fig.5.6) has inspired us to reveal the unknown beta band head energies. For those nuclei whose $2^{+'}$ levels are not known, we have calculated approximate values of γ as explained in previous section. Using these values of $B(E2: I+2 \rightarrow I)_{DR}$ were obtained from the Figs.5.1 to 5.6, in these nuclei. Now using equation (5.9) and these $B(E2: I+2 \rightarrow I)_{DR}$, the values of $B(E2: 2^{+} \rightarrow 0^{+})_{DR}$ were evaluated for different transitions. These values are

TABLE - 5.2

Probable values of Meanlives of Rotational Levels and B(E2)s of Gamma-Ray Cascades.

Nucleus	Transition	γ (degree)	$\frac{e^2 Q_o^2}{16\pi}$	Level Energy(E) (MeV)	$B(E2)_{emp}$ ($e^2.b^2$)	Meanlife τ (p.sec.)
^{102}Mo	$6^+ \rightarrow 4^+$	18.5	0.218	1.305	0.342	4.25
	$8^+ \rightarrow 6^+$			1.957	0.404	1.71
^{104}Mo	$4^+ \rightarrow 2^+$	14.0	0.206	0.561	0.28	42.77
	$6^+ \rightarrow 4^+$			1.081	0.302	2.70
^{106}Mo	$4^+ \rightarrow 2^+$	(13.3)	0.251	0.5225	0.343	44.78
	$6^+ \rightarrow 4^+$			1.034	0.366	6.35
^{98}Ru	$6^+ \rightarrow 4^+$	27.0	0.0815	2.2227	0.157	1.36
	$8^+ \rightarrow 6^+$			3.1267	0.185	0.73
	$10^+ \rightarrow 8^+$			4.001	0.139	1.15
^{100}Ru	$6^+ \rightarrow 4^+$	24.5	0.1077	2.0777	0.194	9.41
	$8^+ \rightarrow 6^+$			3.0629	0.229	0.384
	$10^+ \rightarrow 8^+$			4.086	0.180	0.405
^{102}Ru	$6^+ \rightarrow 4^+$	25.5	0.1333	1.875	0.247	1.23
^{108}Ru	$4^+ \rightarrow 2^+$	22.5	0.185	0.6654	0.279	19.44
^{110}Ru	$4^{\pm} \rightarrow 2^+$	(16.7)	0.1965	0.6639	0.269	22.37
	$6^+ \rightarrow 4^+$			1.24	0.30	4.28
	$8^+ \rightarrow 6^+$			1.9477	0.357	1.287

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^{112}Ru	$4^+ \to 2^+$	(16.7)	0.227	0.6457	0.312	22.88
^{102}Pd	$6^+ \to 4^+$	23.0	0.895	2.1112	0.165	1.22
	$8^+ \to 6^+$			3.0129	0.192	0.71
	$10^+ \to 8^+$			3.9926	0.16	0.56
^{104}Pd	$6^+ \to 4^+$	25.0	0.1082	2.2495	0.198	6.06
	$8^+ \to 6^+$			3.2207	0.235	0.402
	$10^+ \to 8^+$			4.023	0.181	1.35
^{106}Pd	$6^+ \to 4^+$	26.5	0.1276	2.0761	0.244	0.76
	$8^+ \to 6^+$			2.9625	0.287	0.52
^{108}Pd	$6^+ \to 4^+$	27.0	0.1494	1.7713	0.291	1.44
^{110}Pd	$6^+ \to 4^+$	27.0	0.1746	1.5739	0.339	2.02
^{114}Pd	$4^+ \to 2^+$	(21.6)	0.069	0.8536	0.098	21.75
	$6^+ \to 4^+$			1.5029	0.118	5.99
^{116}Pd	$4^+ \to 2^+$	(24.5)	0.116	0.8786	0.169	10.71
^{108}Cd	$6^+ \to 4^+$	24.5	0.0868	2.5413	0.156	0.446
	$8^+ \to 6^+$			3.734	0.184	0.184
^{110}Cd	$4^+ \to 2^+$	26.0	0.1037	1.5425	0.130	1.16
	$6^+ \to 4^+$			2.4798	0.160	7.05
	$8^+ \to 6^+$			3.275	0.189	1.36
	$10^+ \to 8^+$			3.61	0.144	134.3
^{112}Cd	$6^+ \to 4^+$	27.0	0.1098	2.167	0.214	1.56
	$8^+ \to 6^+$			2.8801	0.251	1.76
^{116}Cd	$6^+ \to 4^+$	25.5	0.1224	2.0277	0.226	1.05
^{120}Te	$4^+ \to 2^+$	30.0	0.105	1.1613	0.172	6.05

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	$6^+ \to 4^+$			1.7751	0.223	4.18
	$8^+ \to 6^+$			2.653	0.259	0.605
^{122}Te	$6^+ \to 4^+$	26.2	0.1359	1.750	0.258	5.27
	$8^+ \to 6^+$			2.669	0.304	0.41
	$10^+ \to 8^+$			3.29	0.232	3.8
^{120}Xe	$6^+ \to 4^+$	23.5	0.189	1.3972	0.334	3.11
	$8^+ \to 6^+$			2.0996	0.391	1.22
	$10^+ \to 8^+$			2.871	0.315	0.94
^{122}Xe	$8^+ \to 6^+$	24.0	0.229	2.2184	0.482	0.71
	$10^+ \to 8^+$			3.036	0.382	0.58
^{124}Xe	$4^+ \to 2^+$	25.0	0.18	0.8787	0.266	7.71
	$6^+ \to 4^+$			1.5482	0.329	1.84
	$8^+ \to 6^+$			2.3307	0.391	0.71
^{126}Xe	$4^+ \to 2^+$	26.0	0.154	0.9416	0.234	6.72
	$6^+ \to 4^+$			1.6346	0.290	1.76
	$8^+ \to 6^+$			2.4335	0.343	0.72
	$10^+ \to 8^+$			3.315	0.26	0.59
^{128}Xe	$4^+ \to 2^+$	26.5	0.137	1.0327	0.209	5.47
	$6^+ \to 4^+$			1.737	0.262	1.78
	$8^+ \to 6^+$			2.513	0.308	0.94
^{130}Ba	$4^+ \to 2^+$	24.0	0.272	0.9017	0.359	4.75
	$6^+ \to 4^+$			1.5928	0.444	1.16
	$8^+ \to 6^+$			2.396	0.522	0.47
	$10^+ \to 8^+$			3.365	0.414	0.23

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^{132}Ba	$4^+ \rightarrow 2^+$	26.0	0.146	1.1276	0.218	2.92
	$6^+ \rightarrow 4^+$			1.9328	0.272	0.887
	$8^+ \rightarrow 6^+$			2.796	0.321	0.53
	$10^+ \rightarrow 8^+$			3.115	0.244	106.1
^{136}Ba	$4^+ \rightarrow 2^+$	30.0	0.082	1.866	0.137	0.47
	$6^+ \rightarrow 4^+$			2.2063	0.174	103.7
^{138}Ba	$4^+ \rightarrow 2^+$	30.0	0.0426	1.8987	0.0705	54.25
	$6^+ \rightarrow 4^+$			2.0907	0.089	2916.5
^{132}Ce	$8^+ \rightarrow 6^+$	24.5	0.329	2.3268	0.719	0.37
	$10^+ \rightarrow 8^+$			3.1545	0.563	0.37
^{134}Ce	$8^+ \rightarrow 6^+$	25.5	0.214	2.8117	0.471	0.226
^{142}Ce	$4^+ \rightarrow 2^+$	25.0	0.0911	1.2193	0.135	9.31
^{146}Ce	$4^+ \rightarrow 2^+$	21.5	0.173	0.6682	0.244	28.85
	$6^+ \rightarrow 4^+$			1.1711	0.296	8.5
^{148}Ce	$4^+ \rightarrow 2^+$	(15.5)	0.368	0.4544	0.502	69.0
	$6^+ \rightarrow 4^+$			0.8409	0.552	17.1
^{150}Ce	$4^+ \rightarrow 2^+$	(11.5)	0.755	0.308	1.19	136.8
	$6^+ \rightarrow 4^+$			0.6087	1.072	30.96
	$8^+ \rightarrow 6^+$			0.9851	1.289	8.38
^{146}Nd	$4^+ \rightarrow 2^+$	(21.5)	0.146	1.0434	0.208	5.51
^{148}Nd	$4^+ \rightarrow 2^+$	19.5	0.274	0.751	0.384	11.6
	$6^+ \rightarrow 4^+$			1.355	0.443	2.28
^{154}Nd	$4^+ \rightarrow 2^+$	(10.1)	1.012	0.2352	1.41	352.1
	$6^+ \rightarrow 4^+$			0.4789	1.426	53.86
	$8^+ \rightarrow 6^+$			0.807	1.702	12.6

Contd.......

^{148}Sm	$6^+ \rightarrow 4^+$	23.7	0.1471	1.9061	0.261	1.55
	$8^+ \rightarrow 6^+$			2.5453	0.313	2.48
	$10^+ \rightarrow 8^+$			3.16	0.239	3.89
^{150}Sm	$6^+ \rightarrow 4^+$	(20.5)	0.2828	1.2788	0.475	5.20
	$8^+ \rightarrow 6^+$			1.8371	0.564	2.76
	$10^+ \rightarrow 8^+$			2.4328	0.487	2.23
	$12^+ \rightarrow 10^+$			3.048	0.406	2.28
^{152}Sm	$12^+ \rightarrow 10^+$	13.0	0.6863	2.158	0.90	1.79
^{152}Gd	$4^+ \rightarrow 2^+$	21.5	0.394	0.755	0.557	12.5
	$6^+ \rightarrow 4^+$			1.2274	0.672	5.17
	$8^+ \rightarrow 6^+$			1.747	0.76	2.83
	$10^+ \rightarrow 8^+$			2.3	0.648	2.43
	$12^+ \rightarrow 10^+$			2.884	0.548	2.19
^{158}Gd	$6^+ \rightarrow 4^+$	10.5	0.978	0.539	1.378	32.7
^{160}Gd	$4^+ \rightarrow 2^+$	11.0	1.05	0.2482	1.45	272.8
	$6^+ \rightarrow 4^+$			0.514	1.479	37.8
	$8^+ \rightarrow 6^+$			0.868	1.789	8.2
^{156}Dy	$4^+ \rightarrow 2^+$	15.5	0.753	0.4041	1.04	53.4
	$6^+ \rightarrow 4^+$			0.7705	1.131	10.92
	$8^+ \rightarrow 6^+$			1.2158	1.349	3.45
	$10^+ \rightarrow 8^+$			1.7249	1.28	1.85
	$12^+ \rightarrow 10^+$			2.285	1.12	1.31
^{158}Dy	$6^+ \rightarrow 4^+$	13.0	1.013	0.6378	1.474	16.34
	$8^+ \rightarrow 6^+$			1.0441	1.755	4.19
	$10^+ \rightarrow 8^+$			1.5195	1.75	1.91

Contd........

^{156}Er	$8^+ \rightarrow 6^+$	23.5	0.33	1.9589	0.67	1.34
	$10^+ \rightarrow 8^+$			2.633	0.55	0.95
	$12^+ \rightarrow 10^+$			3.315	0.46	1.2
^{160}Er	$12^+ \rightarrow 10^+$	15.0	0.84	2.3395	1.26	0.99
^{162}Er	$4^+ \rightarrow 2^+$	13.3	1.013	0.3295	1.33	84.4
	$6^+ \rightarrow 4^+$			0.6670	1.433	13.0
	$8^+ \rightarrow 6^+$			1.0968	1.737	3.2
^{164}Er	$6^+ \rightarrow 4^+$	13.0	1.186	0.6144	1.727	4.72
^{168}Er	$6^+ \rightarrow 4^+$	12.0	1.195	0.5487	1.719	23.54
^{170}Er	$6^+ \rightarrow 4^+$	11.5	1.203	0.5411	1.711	25.20
^{162}Yb	$10^+ \rightarrow 8^+$	17.0	0.746	2.0233	1.247	1.01
	$12^+ \rightarrow 10^+$			2.6338	1.088	0.88
^{168}Yb	$4^+ \rightarrow 2^+$	11.8	1.08	0.2865	1.50	136.2
	$6^+ \rightarrow 4^+$			0.5853	1.559	20.77
	$8^+ \rightarrow 6^+$			0.97	1.865	5.19
	$10^+ \rightarrow 8^+$			1.4254	1.894	2.20
	$12^+ \rightarrow 10^+$			1.936	1.718	1.37
^{170}Yb	$4^+ \rightarrow 2^+$	10.8	1.04	0.2774	1.44	163.8
	$6^+ \rightarrow 4^+$			0.5731	1.475	23.12
^{172}Hf	$4^+ \rightarrow 2^+$	12.0	0.928	0.3093	1.27	54.07
	$6^+ \rightarrow 4^+$			0.6281	1.33	18.63
	$8^+ \rightarrow 6^+$			1.0342	1.577	4.68
	$10^+ \rightarrow 8^+$			1.521	1.616	1.905
	$12^+ \rightarrow 10^+$			2.0645	1.453	1.18

Contd........

^{174}Hf	$4^+ \rightarrow 2^+$	10.8	0.9626	0.2974	1.33	125.99
	$6^+ \rightarrow 4^+$			0.6084	1.367	20.55
	$8^+ \rightarrow 6^+$			1.0094	1.629	4.83
	$10^+ \rightarrow 8^+$			1.4856	1.698	1.962
	$12^+ \rightarrow 10^+$			2.02	1.536	1.22
^{176}Hf	$4^+ \rightarrow 2^+$	10.25	1.235	0.2902	1.71	109.68
	$6^+ \rightarrow 4^+$			0.597	1.739	17.26
	$8^+ \rightarrow 6^+$			0.998	2.089	3.77
	$10^+ \rightarrow 8^+$			1.4813	2.182	1.418
	$12^+ \rightarrow 10^+$			2.0349	2.008	0.78
^{178}Hf	$4^+ \rightarrow 2^+$	11.5	1.021	0.3066	1.40	102.89
	$6^+ \rightarrow 4^+$			0.6322	1.448	15.42
	$8^+ \rightarrow 6^+$			1.0585	1.731	3.34
	$10^+ \rightarrow 8^+$			1.571	1.776	1.301
	$12^+ \rightarrow 10^+$			2.1507	1.614	0.77
^{180}Hf	$6^+ \rightarrow 4^+$	10.7	0.974	0.6408	1.385	14.54
	$8^+ \rightarrow 6^+$			1.0839	1.661	2.88
^{180}W	$4^+ \rightarrow 2^+$	14.0	0.863	0.3379	1.18	19.28
	$6^+ \rightarrow 4^+$			0.6888	1.264	12.13
	$8^+ \rightarrow 6^+$			1.1391	1.512	2.91
	$10^+ \rightarrow 8^+$			1.665	1.473	1.378
	$12^+ \rightarrow 10^+$			2.237	1.316	1.01
^{182}W	$8^+ \rightarrow 6^+$	11.5	1.238	1.1445	1.806	2.10
	$10^+ \rightarrow 8^+$			1.7121	1.851	0.748
	$12^+ \rightarrow 10^+$			2.373	1.677	0.386

Contd........

^{182}Os	$4^+ \rightarrow 2^+$	14.8	0.702	0.4002	0.97	48.61
	$6^+ \rightarrow 4^+$			0.7932	1.045	8.28
	$8^+ \rightarrow 6^+$			1.2773	1.237	2.48
	$10^+ \rightarrow 8^+$			1.811	1.187	1.586
	$12^+ \rightarrow 10^+$			2.345	1.053	1.78
^{184}Os	$6^+ \rightarrow 4^+$	14.0	0.569	0.7741	0.833	10.80
	$8^+ \rightarrow 6^+$			1.2749	0.997	2.60
	$10^+ \rightarrow 8^+$			1.870	0.965	1.133
	$12^+ \rightarrow 10^+$			2.546	0.865	0.668
^{186}Os	$8^+ \rightarrow 6^+$	16.5	0.667	1.4209	1.217	1.31
	$10^+ \rightarrow 8^+$			2.069	1.130	0.632
	$12^+ \rightarrow 10^+$			2.782	0.982	0.45
^{188}Os	$6^+ \rightarrow 4^+$	19.0	0.617	0.9398	0.98	3.9
	$8^+ \rightarrow 6^+$			1.5147	1.026	1.28
	$10^+ \rightarrow 8^+$			2.170	2.079	0.325
^{190}Os	$8^+ \rightarrow 6^+$	22.0	0.543	1.6664	1.058	0.87
^{192}Os	$8^+ \rightarrow 6^+$	25.5	0.419	1.7081	1.00	0.89
^{184}Pt	$4^+ \rightarrow 2^+$	19.5	1.07	0.4365	1.48	31.98
	$6^+ \rightarrow 4^+$			0.7981	1.716	7.69
	$8^+ \rightarrow 6^+$			1.2310	1.989	2.73
	$10^+ \rightarrow 8^+$			1.7074	1.753	1.89
	$12^+ \rightarrow 10^+$			2.205	1.492	1.79
^{186}Pt	$4^+ \rightarrow 2^+$	21.5	0.71	0.4901	0.987	31.66
	$6^+ \rightarrow 4^+$			0.8772	1.206	7.78
	$8^+ \rightarrow 6^+$			1.3411	1.368	2.78

Contd........

	$10^+ \to 8^+$			1.857	1.165	1.93
	$12^+ \to 10^+$			2.335	0.983	3.33
^{188}Pt	$4^+ \to 2^+$	26.5	0.52	0.6706	0.82	9.13
	$6^+ \to 4^+$			1.1846	1.00	2.27
	$8^+ \to 6^+$			1.7823	1.18	0.906
	$10^+ \to 8^+$			2.438	0.894	0.76
	$12^+ \to 10^+$			2.676	0.724	29.24
^{190}Pt	$4^+ \to 2^+$	28.5	0.52	0.7369	0.83	5.83
	$6^+ \to 4^+$			1.2875	1.058	1.52
	$8^+ \to 6^+$			1.9151	1.25	0.67
	$10^+ \to 8^+$			2.5353	0.906	0.98
	$12^+ \to 10^+$			2.7267	0.739	335.2
^{228}Th	$6^+ \to 4^+$	9.8	1.41	0.3782	1.967	107.94
^{230}Th	$6^+ \to 4^+$	10.5	1.625	0.357	2.285	115.55
	$8^+ \to 6^+$			0.592	2.735	30.27
^{232}U	$4^+ \to 2^+$	9.4	1.98	0.1566	2.76	183.88
	$6^+ \to 4^+$			0.3216	2.757	91.15
	$8^+ \to 6^+$			0.5406	3.307	31.20
	$10^+ \to 8^+$			0.805	3.521	15.71
	$12^+ \to 10^+$			1.112	3.24	8.55
^{234}U	$4^+ \to 2^+$	8.7	2.053	0.1433	2.83	278.70
	$6^+ \to 4^+$			0.2961	2.783	130.80
	$8^+ \to 6^+$			0.4971	3.40	46.7
	$10^+ \to 8^+$			0.741	3.618	20.86
	$12^+ \to 10^+$			1.024	3.358	11.95

Contd.........

^{238}U	$4^+ \rightarrow 2^+$	8.3	2.395	0.1484	3.33	197.44
	$6^+ \rightarrow 4^+$			0.3072	3.239	93.96
	$8^+ \rightarrow 6^+$			0.5183	3.878	32.23
	$10^+ \rightarrow 8^+$			0.7757	4.255	14.01
	$12^+ \rightarrow 10^+$			1.0765	3.972	7.72
^{238}Pu	$4^+ \rightarrow 2^+$	8.3	2.52	0.1460	3.51	172.65
	$6^+ \rightarrow 4^+$			0.3036	3.425	84.50
	$8^+ \rightarrow 6^+$			0.514	4.09	29.78
^{240}Pu	$4^+ \rightarrow 2^+$	8.6	2.536	0.1417	3.52	135.72
	$6^+ \rightarrow 4^+$			0.2943	3.453	95.19
	$8^+ \rightarrow 6^+$			0.4976	4.137	34.47
	$10^+ \rightarrow 8^+$			0.751	4.514	13.52
^{242}Pu	$4^+ \rightarrow 2^+$	8.15	2.711	0.1495	3.76	145.61
	$6^+ \rightarrow 4^+$			0.3097	3.643	75.82
	$8^+ \rightarrow 6^+$			0.5227	4.273	26.16
	$10^+ \rightarrow 8^+$			0.779	4.806	11.04
	$12^+ \rightarrow 10^+$			1.087	4.529	6.50
^{244}Pu	$4^+ \rightarrow 2^+$	8.5	2.799	0.154	3.87	117.15
	$6^+ \rightarrow 4^+$			0.315	3.791	71.06
	$8^+ \rightarrow 6^+$			0.537	4.527	20.89
^{246}Cm	$4^+ \rightarrow 2^+$	7.8	3.045	0.1420	4.21	102.09
	$6^+ \rightarrow 4^+$			0.2956	4.083	77.92
	$8^+ \rightarrow 6^+$			0.5004	4.86	27.25

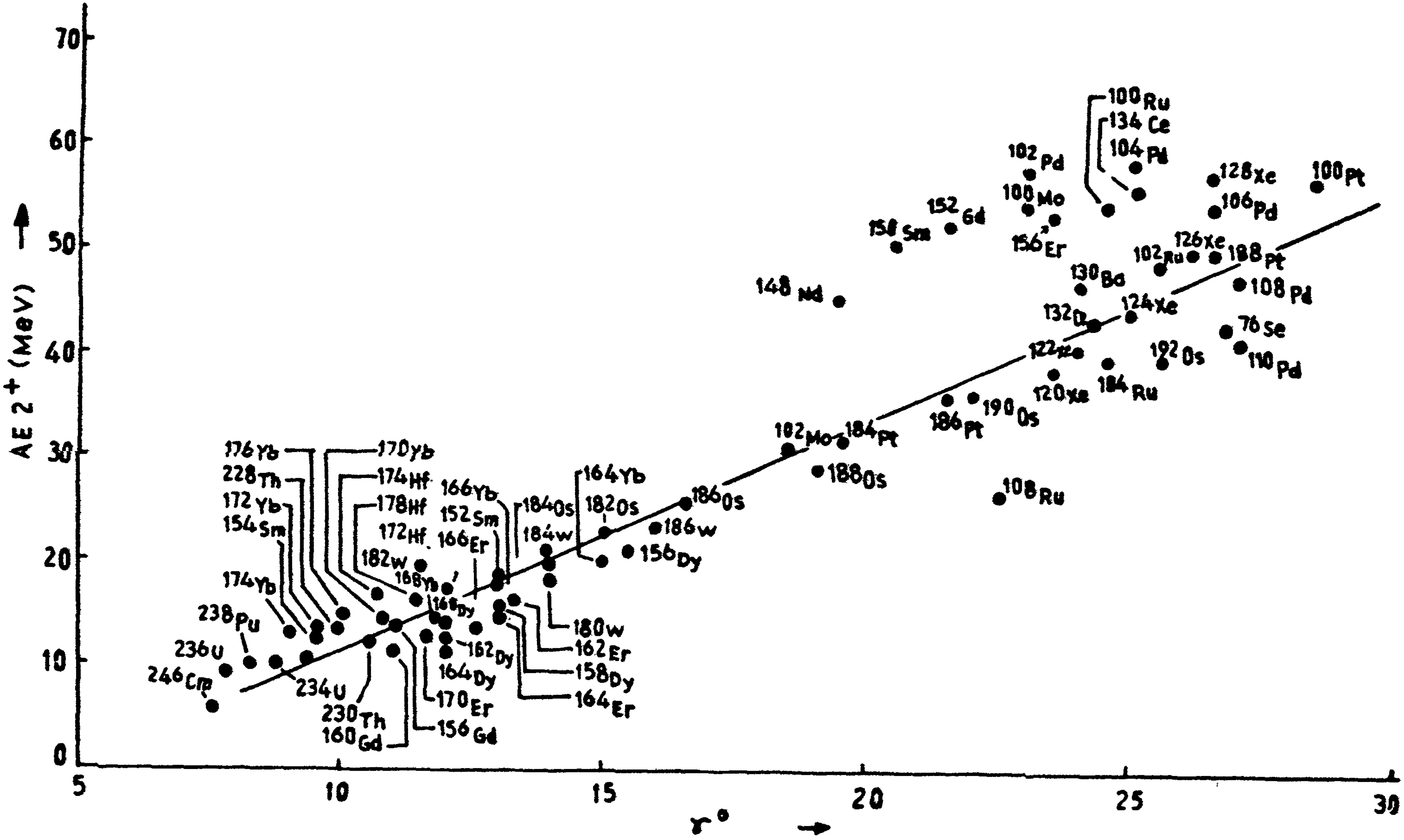

Fig. 5.7 A plot of product of the energy of the first excited state and the mass number against non-axiality parameter γ.

tabulated in Table 5.3. The values under column a,b,c,d, e are the $B(E2: 2^+ \rightarrow 0^+)_{DR}$ values obtained from F_{DR} plotted for $6^+ \rightarrow 4^+$, $8^+ \rightarrow 6^+$, $10^+ \rightarrow 8^+$, $12^+ \rightarrow 10^+$, $14^+ \rightarrow 12^+$ transitions. In the last column of this table the mean value of $B(E2:2^+ \rightarrow 0^+)_{DR}$ are shown. Using approximate γ values obtained from fig. 5.7 and DF relation [equation (2.39)], $E2^{+\prime}$ values were predicted. These $E2^{+\prime}$ values were used for the calculation of s. Now using values of $B(E2: 2^+ \rightarrow 0^+)_{DR}$, s, Q_0 and supplying Clebsch-Gordan Coefficients in equation 5.9, the values of q were calculated which give beta band head energy $E0^{+\prime}$, since

$$E0^{+\prime} = q\, E2^{+}$$

These predicted values of $E0^{+\prime}$ are given in Table 5.4.

Our results of $E0^{+\prime}$ for ^{176}Yb, ^{166}Hf, ^{168}Hf and ^{192}Os nuclei are very close to their respective suspected values given in Table of Isotopes[20]. This agreement within an error of 5% confirms the experimentally observed but suspected levels as $E0^{+\prime}$ for these nuclei

5.3 Conclusion:

With the development of new experimental techniques, high energy heavy ions carry large angular momentum, which the compound nucleus can not get rid of by evaporation of neutrons. Therefore, high spin states result in the final nucleus and the corresponding gamma-ray cascades have been observed. In even-even nuclei several cascades interpreted

TABLE - 5.3

$B(E2)_{DR}$ Values in Units of $e^2.b^2$ calculated from Figures 5.2 to 5.6.

Nucleus	γ (degree)	$B(E\,2{:}\;2^+ \rightarrow 0^+)_{DR}$ (a)	(b)	(c)	(d)	(e)	(f)
^{164}Dy	12.0	0.814 (32)	0.735 (48)	0.869 (6)	0.987 (17)	0.964 (52)	0.874 (31)
^{160}Yb	(22.5)	0.248 (25)	0.172 (43)	0.465 (128)	-	-	0.295 (65)
^{162}Yb	17.0	0.407 (76)	0.310 (79)	-	-	-	0.358 (77)
^{176}Yb	10.25	0.807 (102)	-	0.940 (77)	0.952 (87)	0.947 (178)	0.911(111)
^{166}Hf	16.7	0.432 (63)	0.401 (88)	0.591	-	-	0.475(125)
^{168}Hf	(14.3)	0.547 (57)	0.510 (48)	0.638 (94)	-	0.694 (155)	0.597 (88)
^{192}Os	25.5	0.210 (32)	-	-	-	-	0.210 (32)

as coming from high spin states have been identified. We notice that the energy spacing between higher levels in different nuclei is almost constant for the same value of I varying from 8 to 16. This indicates that the moments of inertia of the rotating nuclei in these high spin states become almost the same. The centrifugal stretching increases the deformation, pairing becomes less important and the nucleus behaves as a rigid rotor. Retaining an almost spheroidal shape, but allowing stretching, leaves only one parameter γ free to describe the change in the moment of inertia in going up the band. Extremely good agreement has been obtained with variation of this parameter. This has obviously encouraged us to predict B(E2) values, and meanlives τ, using one parameter dependent empirical relation. However there is a risk of not getting rotational band at high spin states since if more and more pairs were broken up, the wave function of higher spin states no longer resembles that of the ground state and the rotational band breaks off[24]. So far there is no such example of rotational band break off, although the limit predicted by Mottelson et al.[24], viz, I = 14 to 18, has already been passed in some cases. For the lighter nucleus such as Xe, this limit lies at the spin 10 or 12.

We have compared our predictions of B(E2) values and meanlives with their respective experimental values, if

TABLE - 5.4

Predictions of Beta Band Head Energies. Bracketted Values are our Predictions.

Nucleus	$\frac{e^2 Q_o^2}{16\pi}$	$E2^{+'}$ (keV)	s	q	Predicted $E0^{+'}$(keV)	Suspected[+] $E0^{+'}$(keV)
^{164}Dy	1.18	761.8	10.38	(9.36)	(687)	-
^{160}Yb	0.462	(713)	(2.93)	(11.05)	(2685)	-
^{162}Yb	0.746	875	5.26	(3.90)	(648.5)	-
^{176}Yb	1.05	1261	15.35	(16.63)	(1365.8)	1340
^{166}Hf	0.86	(869)	(5.47)	(4.53)	(718.9)	695
^{168}Hf	0.836	(940)	(7.60)	(7.38)	(912.9)	940
^{192}Os	0.420	489.0	2.38	(4.69)	(965)	956

[+]Values taken from reference 20.

known, and have found an excellent agreement for all nuclei except ^{134}Ba, ^{134}Ce and ^{140}Ce. It appears that the disagreement between the predictions of meanlives and their respective experimental values for the nuclei ^{134}Ba and ^{134}Ce, is due to the neutron deficiency and consequently there is a significantly influence of kinetic energy on the collective motion in the γ-direction[25]. A special deformation dependence of kinetic energy can also privilege the deformation and make the wave function localize around it, which makes the deformation dynamic. This is why the rigid rotor model has failed to reproduce the experimental findings for these particular nuclei. The experimental fact that first excited 0^+ state ($E0^{+'}$ = 1760 keV) is not well seperated from first excited 2^+ state ($E2^+$ = 604 keV) at higher energies for ^{134}Ba nucleus, also supports this view point[4]. The discrepancy in results of ^{140}Ce is due to the fact that it has neutron number 82 which is magic.

Our study lends support to the assumption of tri-axial shape by Davydov and Chaban[26] and Davydov-Rostovsky[14] and confirms experimentally observed $K^{\pi} = 0^+$ levels with energy of about 1 MeV as K = 0 components of the quadrupole shape oscillation which are interpreted as classical beta vibrations in contrast to Zawischa et al.[27] who doubted the collective nature of low-lying levels and suggested that only high-lying $K^{\pi}=0^+$ resonances are classical beta vibrations.

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CHAPTER - VI

MEANLIFE PREDICTIONS FOR $K^{\pi} = 2^{+}$ COLLECTIVE EXCITATIONS

6.1 Introduction:

The study of absolute B(E2) values and B(E2) branching ratios for the transitions depopulating 2^{+} state of the gamma vibrational band has been done by some workers[1-4]. Their work is limited to small values of non-axiality parameter γ and to few nuclei in rare earth and actinide regions. Therefore the systematics drawn by them do not reflect the characteristic properties of Asymmetric Rotor Model (ARM). Thus the inferences drawn by previous workers can not be considered much reliable.

Augusthy et al.[1] have calculated $B(E2: 2^{+\prime} \rightarrow 0^{+})$ and $B(E2: 2^{+\prime} \rightarrow 2^{+})$ values by employing the theoretical deformation parameter[5]. Puri et al[2] have favoured the Davydov-Filippov (DF)[6] model so far as the mixing ratio δ (E2/M1 mixing ratio, which provides rather a sensitive test of nuclear wave functions) are concerned and the Davydov-Rostovsky[7] (DR) model so far as the B(E2) ratios are concerned, in their study of the properties of 2^{+} level of gamma vibrational band in $150 < A < 190$ region. They observed that experimental δ values are much closer to DF values than to the pairing force model of Greiner[8] or the coriolis interaction model of Bes et al.[9]. The lead of DR model over DF model for B(E2) branching ratios shown by Puri et al. has no justification since they have taken non-axiality parameter γ suggested by Varshni et al.[10] which has already been criticised in Chapter IV.

Agarwal[3] used the intrinsic electric quadrupole moment Q_o derived from experimental B(E2) values compiled by Stelson et al.[11] which are now outdated and have changed to much extent. Gupta et al.[5] in their study of absolute B(E2) values and B(E2) branching ratios for the transitions depopulating 2^+ state of gamma vibrational band and the inter rotational band in deformed even heavy mass nuclei, confirmed that low-lying $K^{\pi} = 2^+$ resonances are classical gamma vibrations. Their conclusion differed with that of Zawischa et al.[12] who doubted the collective nature of low-lying $K^{\pi} = 2^+$ resonances and indicated that only high lying $K^{\pi} = 2^+$ resonances are classical gamma vibrations. In their study Gupta et al. have not considered deformed medium mass nuclei having large values of non-axiality parameter γ. Hence conclusion drawn by them are not wide based.

The medium mass nuclei were studied long ago by Tamura et al.[13], but due to fast growth in experimental techniques, the results have been revised significantly. In addition many new transitions from $2^{+\prime}$ to 2^+ and from $2^{+\prime}$ to 0^+ and branching ratio $B(E2: 2^{+\prime} \rightarrow 2^+/0^+)$ have become known. Recently Kishimoto et al.[14] and Weeks et al.[15] applying Boson-expansion description of collective states in ^{114}Cd, ^{122}Te and in even isotopes of Ru and Pd tried to explain partially the absolute B(E2) values and branching ratios.

Since the new data obtained from heavy ion experiments have supported the DF model and indicated that a large number of heavy mass transitional nuclei have tri-axial shapes which are more stable than expected from theoretical potential energy surface calculations of Baranger and Kumar[16]. So we thought it worthwhile to consider the nuclei covering both the medium as well as the heavy mass regions. The results of Gupta et al.[5] have been modified by including a large number of medium mass nuclei having their γ values between 15° and 25°. It is also more promising to attempt the Asymmetric Rotor Model (ARM) in the region with $\gamma = 20^{\circ}$ or so, since there is no very general theoretical argument against ARM in this region. In our calculations we have taken ARM dependent intrinsic quadrupole moment Q_0 given by

$$e^2 Q_0^2 = 16\pi \, [B(E2: 2^+ \rightarrow 0^+) + B(E2: 2^{+\prime} \rightarrow 0^+)] \qquad [6.1]$$

In case where second part of equation (6.1) is not known only the first part is taken into calculations. We have predicted meanlives of $2^{+\prime}$ states from the predicted values of $B(E2: 2^{+\prime} \rightarrow 0^+)$ and $B(E2: 2^{+\prime} \rightarrow 2^+)$ which were obtained on feeding absolute $B(E2: 2^+ \rightarrow 0^+)$ and γ, for a large number of nuclei.

6.2 Method of Calculation:

The non-axiality parameter γ has been computed from the relation[6)]

$$\frac{E2^{+\prime}}{E2^{+}} = \frac{1 + [\,1 - \frac{8}{9}\,\mathrm{Sin}^2\,(3\gamma)]^{1/2}}{1 - [\,1 - \frac{8}{9}\,\mathrm{Sin}^2\,(3\gamma)]^{1/2}} \qquad [6.2]$$

where $E2^{+\prime}$ and $E2^{+}$ are the experimental values of the energies of second 2^+ and first 2^+ excited states.

The reduced transition probability estimates (DF) are calculated from following relations[6)] expressed in units of $e^2Q_o^2/16\pi$.

$$B(E2: 2^{+\prime} \rightarrow 0^+)_{DF} = \frac{1}{2}\left[\,1 - \frac{3 - 2\,\mathrm{Sin}^2(3\gamma)}{[9 - 8\,\mathrm{Sin}^2(3\gamma)]^{1/2}}\,\right] \qquad [6.3]$$

$$B(E2: 2^{+\prime} \rightarrow 2^+)_{DF} = \frac{10}{7}\left[\frac{\mathrm{Sin}^2\,(3\gamma)}{9 - 8\,\mathrm{Sin}^2(3\gamma)}\right] \qquad [6.4]$$

$$B(E2: 2^{+} \rightarrow 0^+)_{DF} = \frac{1}{2}\left[\,1 + \frac{3 - 2\mathrm{Sin}^2(3\gamma)}{\{9 - 8\mathrm{Sin}^2(3\gamma)\}^{1/2}}\,\right] \qquad [6.5]$$

The reduced transition probability estimates (DR) are calculated from the relations[7)], expressed in units of $e^2Q_o^2/16\pi$ as

$$B(E2:\ 2^{+\prime} \rightarrow 0^+)_{DR} = \frac{1}{s}\left(1 - \frac{3}{2s}\right)\left(1 - \frac{9s}{4q^2}\right)^2 \qquad [6.6]$$

$$B(E2:\ 2^{+\prime} \rightarrow 2^+)_{DR} = \frac{10}{7s}\left(1 + \frac{5}{2s}\right)\left(1 - \frac{9s}{4q^2}\right)^2 \qquad [6.7]$$

$$B(E2:\ 2^{+} \rightarrow 0^+)_{DR} = \left(1 - \frac{1}{s}\right)\left(1 - \frac{2s}{3q^2}\right)^2 \qquad [6.8]$$

where the parameters s and q are expressed as

$$s = \frac{E2^{+\prime}}{E2^+} \quad \text{and} \quad q = \frac{E0^{+\prime}}{E2^+} \qquad [6.9]$$

$E0^{+\prime}$ is the energy of the first 0^+ excited state or the beta band head energy. The values of $E0^{+\prime}$, $E2^{+\prime}$ and $E2^+$ are taken from Sakai and Rester Table[17].

The experimental values of B(E2) have been extracted from meanlives using following relations, in units of $e^2.b^2$

$$B(E2:\ 2^{+} \rightarrow 0^+) = \frac{0.0812}{E_\gamma^5 (1+\alpha_T)\ \tau} \qquad [6.10]$$

$$B(E2:\ 2^{+\prime} \rightarrow 0^+) = \frac{0.0812\ I_x}{E_\gamma^5 (1+\alpha_T)\ \tau} \qquad [6.11]$$

$$B(E2:\ 2^{+\prime} \rightarrow 2^+) = \frac{0.0812\ I_y}{E_\gamma^5 (1+\alpha_T)\ \tau} \cdot \frac{\delta^2}{(1+\delta^2)} \qquad [6.12]$$

where I_x and I_y are the intensities of γ-rays in $2^{+\prime} \rightarrow 0^+$ and $2^{+\prime} \rightarrow 2^+$ transitions and δ is the mixing ratio (E2/M1) and is expressed as

$$\delta^2 = 8.1 \times 10^{-11} \times Z^2 A^{4/3} E_\gamma^2 \qquad [6.13]$$

I_x, I_y, E_γ , the transition energy in MeV, τ , meanlife of excited state in peco sec. and α_T, the total internal conversion coefficients have been taken from Table of Isotopes[18].

6.2(a) Probable B(E2) Values and Meanlives:

Tables 6.1 and 6.2 illustrate the calculations of experimental, DR and DF values of B(E2: $2^+ \rightarrow 0^+$), B(E2: $2^{+\prime} \rightarrow 0^+$) and B(E2: $2^{+\prime} \rightarrow 2^+$) expressed in the units of $e^2.b^2$ with input parameter s, q and $e^2 Q_0^2/16\pi$. The values of γ, for the cases in which $2^{+\prime}$ levels are experimentally not known, have been calculated using the systematic [Fig.5.7] and are expressed in brackets. The uncertainties in the experimental values of B(E2) for these transitions are expressed in paranthesis just after their values. The branching ratios B(E2: $2^{+\prime} \rightarrow 0^+$)/B(E2: $2^+ \rightarrow 0^+$), B(E2: $2^{+\prime} \rightarrow 2^+$)/B(E2: $2^+ \rightarrow 0^+$) and B(E2: $2^{+\prime} \rightarrow 2^+/0^+$) are plotted against non-axiality parameter γ in Figs. 6.1 to 6.3 respectively. Fig. 6.1 and 6.2 are employed to predict B(E2: $2^{+\prime} \rightarrow 0^+$) and B(E2: $2^{+\prime} \rightarrow 2^+$) values on feeding B(E2: $2^+ \rightarrow 0^+$) experimental values for those nuclei having unknown meanlives for $2^{+\prime}$ known levels. The predicted values of B(E2: $2^{+\prime} \rightarrow 0^+$) and meanlives for a large number of nuclei are given in Table 6.3, while predicted values of B(E2: $2^{+\prime} \rightarrow 2^+$) and the respective meanlives are listed in Table 6.4. The last column of Table 6.4 shows the mean values

TABLE - 6.1

$B(E2)_{exp}$, $B(E2)_{DR}$ and $B(E2)_{DF}$ Values with input parameters s,q, $e^2Q_o^2/16\pi$ and γ.

Nucleus	s	q	$B(E2: 2^+ \to 0^+)(e^2.b^2)$ Exptl.	DR	DF	$\frac{e^2Q_o^2}{16\pi}$	γ (degree)
^{74}Ge	2.021	2.488	0.0609 (6)	0.0192	0.0619	0.0622	29.0
^{76}Se	2.175	2.007	0.0908(82)	0.0233	0.1030	0.1053	26.75
^{78}Se	(2.132)	(2.442)	0.0710(64)	0.0222	0.0700	0.0721	27.0
^{80}Se	2.175	2.218	0.0516(46)	0.0146	0.0520	0.0535	27.0
^{98}Mo	2.233	0.933	0.057 (3)	0.016	0.058	0.0597	26.25
^{100}Mo	2.731	1.292	0.1228(122)	0.00056	0.119	0.1263	23.0
^{98}Ru	2.169	2.143	0.080(16)	0.0301	0.0798	0.0815	27.0
^{100}Ru	2.524	2.095	0.104 (7)	0.0238	0.102	0.1077	24.5
^{102}Ru	2.322	1.986	0.130 (9)	0.0091	0.128	0.1333	25.5
^{104}Ru	2.494	(2.760)	0.164 (12)	0.050	0.163	0.1695	24.5
^{102}Pd	2.757	(2.859)	0.090(10)	0.0469	0.089	0.0950	23.0

Contd....

^{104}Pd	2.414	2.399	0.104 (4)	0.0455	0.103	0.1080	25.0
^{106}Pd	2.204	2.215	0.124 (8)	0.0329	0.1246	0.1276	26.5
^{108}Pd	2.146	2.426	0.146 (10)	0.044	0.147	0.1494	27.0
^{110}Pd	2.176	2.533	0.172 (12)	0.055	0.171	0.1746	27.0
^{106}Cd	2.714	-	0.0767 (70)	-	0.079	0.0837	23.5
^{108}Cd	2.539	(2.719)	0.081 (7)	0.0266	0.082	0.0868	24.5
^{110}Cd	2.244	(2.240)	0.085 (1)	0.023	0.082	0.1037	26.0
^{112}Cd	2.134	1.981	0.1080(76)	0.023	0.108	0.1098	27.0
^{114}Cd	2.166	2.031	0.1168(82)	0.0266	0.116	0.1184	27.0
^{116}Cd	2.371	2.497	0.120 (8)	0.038	0.117	0.1224	25.5
^{122}Te	2.229	2.407	0.132 (2)	0.055	0.132	0.1359	26.2
^{124}Te	2.200	2.749	0.114 (2)	0.045	0.117	0.1202	26.5
^{126}Te	2.132	2.819	0.096 (2)	0.034	0.093	0.0961	27.0
^{134}Ba	1.931	2.911	0.140 (3)	0.046	0.140	0.1413	30.0
^{150}Nd	8.165	5.20	0.640 (30)	0.360	0.610	0.669	14.0
^{148}Sm	2.642	2.04	0.141 (5)	0.029	0.139	0.147	23.7
^{150}Sm	3.575	2.22	0.274 (6)	0.049	0.265	0.283	20.5

Contd....

^{152}Sm	8.90	5.62	0.670 (15)	0.4301	0.657	0.686	13.0
^{154}Sm	17.55	13.40	0.922 (40)	0.760	0.911	0.935	9.5
^{152}Gd	3.22	1.79	0.394 (26)	0.029	0.368	0.394	21.5
^{154}Gd	8.09	5.53	0.770 (16)	0.520	0.760	0.799	14.0
^{156}Gd	12.97	11.79	0.915 (10)	0.740	0.918	0.950	11.0
^{158}Gd	14.93	15.04	0.994 (10)	0.850	0.948	0.978	10.5
^{160}Hd	13.13	-	1.030 (10)	-	1.015	1.050	11.0
^{158}Dy	9.56	10.02	0.980 (70)	0.770	0.969	1.013	13.0
^{160}Dy	11.13	14.69	0.998 (60)	0.847	0.961	0.019	12.0
^{162}Dy	11.01	14.02	1.089 (30)	0.917	1.061	1.110	12.0
^{164}Dy	10.38	-	1.140	-	1.095	1.16	12.0
^{156}Er	2.70	2.70	0.330 (20)	0.119	0.312	0.330	23.5
^{158}Er	4.26	4.20	0.560 (30)	0.294	0.520	0.560	18.5
^{160}Er	6.80	7.11	0.840 (40)	0.595	0.795	0.840	15.0
^{162}Er	8.82	10.65	0.976 (49)	0.778	0.933	1.013	13.3
^{164}Er	9.41	13.63	1.150 (70)	0.959	1.135	1.186	13.0
^{166}Er	9.75	18.12	1.122 (40)	0.967	1.076	1.148	12.5

Contd....

^{168}Er	10.29	15.25	1.170 (40)	1.000	1.150	1.195	12.0
^{170}Er	11.86	11.33	1.185 (30)	0.960	1.160	1.203	11.5
^{164}Yb	7.01	7.91	0.930 (35)	0.68	0.880	0.930	15.0
^{166}Yb	9.11	10.19	1.060 (150)	0.810	1.010	1.060	13.0
^{168}Yb	11.21	13.18	1.080 (50)	0.906	1.050	1.080	11.8
^{170}Yb	13.51	12.69	1.040 (20)	0.861	1.010	1.040	10.8
^{172}Yb	18.61	13.24	1.186 (50)	0.970	1.160	1.192	9.5
^{174}Yb	21.36	19.45	1.148 (56)	1.010	1.130	1.156	9.0
^{176}Yb	15.35	-	1.050	-	1.030	1.062	10.0
^{174}Hf	13.48	9.10	0.935 (44)	0.710	0.932	0.963	10.8
^{176}Hf	15.18	13.01	1.215 (35)	1.02	1.20	1.235	10.25
^{178}Hf	12.61	12.87	0.998	0.85	0.98	1.021	11.5
^{180}Hf	13.93	11.86	0.974 (30)	0.55	0.94	0.974	10.7
^{182}W	12.20	11.34	1.029 (11)	0.85	1.02	1.238	11.5
^{184}W	8.12	9.01	0.667 (12)	0.54	0.66	1.093	14.0
^{186}W	6.03	7.21	0.669 (9)	0.46	0.66	1.012	16.0
^{186}Os	5.59	7.74	0.630 (6)	0.45	0.63	0.667	16.5

Contd....131..

^{188}Os	4.08	7.00	0.568 (6)	0.37	0.58	0.617	19.0
^{190}Os	2.99	4.88	0.496 (4)	0.27	0.51	0.543	20.0
^{192}Os	2.38	-	0.420 (4)	-	0.44	0.459	25.5
^{230}Th	14.69	11.94	1.600 (400)	1.29	1.57	1.625	10.5
^{232}Th	15.90	14.79	1.850 (80)	1.57	1.82	1.870	10.0
^{234}U	21.11	18.62	2.030 (90)	1.78	1.98	2.053	8.7
^{236}U	21.18	20.32	2.320 (80)	2.06	2.27	2.360	8.7
^{238}U	23.60	20.59	2.380 (50)	2.11	2.33	2.395	8.3
^{240}Pu	21.90	20.10	2.530 (70)	2.23	2.47	2.536	8.6
^{242}Pu	24.22	21.01	2.680 (110)	2.381	2.63	2.711	8.15
^{244}Pu	22.06	-	2.770 (70)	-	2.71	2.799	8.5
^{246}Cm	26.24	27.41	3.010 (90)	2.76	2.94	3.045	7.8
^{248}Cm	24.29	29.71	3.000	2.80	2.98	3.036	8.0

Values have been calculated using references 18, 20 and 21.

TABLE - 6.2

$B(E2)_{exp}$, $B(E2)_{DR}$ and $B(E2)_{DF}$ Values for $2^{+'} \rightarrow 0^{+}$ and $2^{+'} \rightarrow 2^{+}$ Transitions.

Nucleus	$B(E2: 2^{+'} \rightarrow 0^{+})$ $(e^2.b^2)$			$B(E2: 2^{+'} \rightarrow 2^{+})$ $(e^2.b^2)$		
	Exptl.	DR	DF	Exptl.	DR	DF
^{74}Ge	0.0013 (5)	5.6×10^{-4}	2.5×10^{-4}	0.038 (14)	0.0069	0.0877
^{76}Se	0.0015 (3)	0.0018	0.0025	0.087 (3)	0.0182	0.1158
^{78}Se	0.0011 (2)	3.8×10^{-4}	0.0015	0.059 (18)	0.0040	0.0822
^{80}Se	0.0019 (4)	2.1×10^{-7}	0.0011	0.078 (36)	1.9×10^{-6}	0.0610
^{98}Mo	0.0027 (4)	0.1998	0.0018	0.166 (23)	1.8431	0.0621
^{100}Mo	0.0035 (5)	0.1473	0.0071	>0.121 (10)	0.8943	0.0854
^{98}Ru	0.0015 (2)	4.5×10^{-5}	0.0017	0.147 (25)	0.0004	0.0929
^{100}Ru	0.0037 (5)	0.0015	0.0059	0.091 (15)	0.0105	0.0885
^{102}Ru	0.0033 (4)	0.0021	0.0049	0.117 (15)	0.0179	0.1244
^{104}Ru	0.0055 (6)	0.0019	0.0079	0.123 (19)	0.0135	0.1396
^{102}Pd	0.0050 (3)	0.0009	0.0053	0.039 (1)	0.0055	0.0642

Contd....

^{104}Pd	0.0042 (3)	5.3×10^{-5}	0.0046	0.084 (18)	0.0004	0.0936
^{106}Pd	0.0036 (3)	$2.1. \times 10^{-6}$	0.0033	0.136 (2)	$2.0. \times 10^{-5}$	0.1360
^{108}Pd	0.0034 (3)	0.0007	0.0031	0.248 (55)	0.0061	0.1703
^{110}Pd	0.0026 (2)	0.0014	0.0037	0.180 (30)	0.0138	0.1990
^{106}Cd	0.0050 (8)	-	0.0045	0.072 (12)	-	0.0610
^{108}Cd	0.0058 (7)	0.0007	0.0041	0.078 (9)	0.0050	0.0714
^{110}Cd	0.0029 (4)	6.0×10^{-7}	0.0033	0.144 (42)	5.5×10^{-6}	0.1058
^{112}Cd	0.0018 (2)	0.0008	0.0025	0.106	0.0080	0.1231
^{114}Cd	0.0016 (1)	0.0005	0.0026	0.079 (23)	0.0055	0.1350
^{116}Cd	0.224 (3)	0.0004	0.0045	0.098 (19)	0.0033	0.1147
^{122}Te	0.0039 (23)	0.0004	0.0041	0.348	0.0033	0.1400
^{124}Te	0.0062 (41)	0.0021	0.0044	0.638 (425)	0.0198	0.1298
^{126}Te	0.0011	0.0021	0.0020	0.350	0.0220	0.1095
^{134}Ba	0.0013 (2)	-	≈ 0	0.207 (24)	-	-
^{150}Nd	0.029 (3)	0.0069	0.031	-	0.0157	0.080
^{148}Sm	0.006 (1)	0.0045	0.008	0.091 (15)	0.0292	0.068
^{150}Sm	0.0088 (20)	0.0186	0.018	0.039 (14)	0.0778	0.102

Contd....134...

^{152}Sm	0.016 (1)	0.0084	0.029	0.042 (4)	0.0184	0.073
^{154}Sm	0.013 (3)	0.0296	0.024	0.020	0.0529	0.044
^{152}Gd	-	0.1049	0.024	-	0.4986	0.184
^{154}Gd	0.028 (2)	0.0131	0.038	0.059	0.3018	0.099
^{156}Gd	0.024 (1)	0.0404	0.031	0.029	0.0799	0.065
^{158}Gd	0.018 (2)	0.0427	0.029	0.018	0.0792	0.059
^{160}Gd	0.020 (1)	-	0.035	-	-	0.072
^{158}Dy	0.033	0.0551	0.043	0.078	0.1177	0.107
^{160}Dy	0.021 (1)	0.0619	0.039	0.040	0.1251	0.089
^{162}Dy	0.021 (1)	0.0665	0.042	0.039	0.1349	0.097
^{164}Dy	0.020 (1)	-	0.044	0.041	-	0.102
^{156}Er	-	0.0015	0.018	-	0.0093	0.241
^{158}Er	-	0.0176	0.035	-	0.0065	0.164
^{160}Er	-	0.0468	0.044	-	0.1174	0.120
^{162}Er	0.037	0.0648	0.044	-	0.1432	0.107
^{164}Er	0.036 (10)	0.0831	0.051	0.080	0.1788	0.126
^{166}Er	0.026 (2)	0.0867	0.046	0.045	0.1840	0.107

Contd....185...

^{168}Er	0.025 (1)	0.0720	0.045	0.045	0.1497	0.105
^{170}Er	0.018 (1)	0.0555	0.042	0.031	0.1102	0.094
^{164}Yb	-	0.0584	0.049	-	0.1441	0.133
^{166}Yb	-	0.0626	0.045	-	0.1365	0.112
^{168}Yb	-	0.0609	0.080	-	0.1229	0.078
^{170}Yb	-	0.0450	0.031	-	0.0857	0.069
^{172}Yb	0.006	0.0341	0.031	0.012	0.0601	0.055
^{174}Yb	0.008	0.0383	0.028	-	0.0658	0.050
^{176}Yb	0.012 (3)	-	0.030	-	-	0.054
^{174}Hf	0.0276 (40)	0.0255	0.031	0.042	0.0496	0.066
^{176}Hf	0.0250 (10)	0.0467	0.036	0.032	0.0863	0.069
^{178}Hf	0.0226 (24)	0.0490	0.036	0.039	0.0952	0.082
^{180}Hf	0.0230	0.0377	0.031	0.032	0.0715	0.062
^{182}W	0.0250 (10)	0.0551	0.043	0.049	0.1081	0.095
^{184}W	0.0280 (10)	0.0659	0.052	0.051	0.1510	01136
^{186}W	0.0280 (10)	0.0689	0.057	0.067	0.1852	0.188
^{186}Os	0.037 (5)	0.0544	0.038	0.082 (12)	0.1528	0.135

Contd...

...

^{188}Os	0.0490 (30)	0.0631	0.040	0.156 (11)	0.2297	0.194
^{190}Os	0.0468 (30)	0.0467	0.032	0.270 (20)	0.2459	0.309
^{192}Os	0.0390 (20)	-	0.017	0.328 (37)	-	0.428
^{230}Th	0.0246	0.0568	0.049	0.066	0.1091	0.097
^{232}Th	0.0244	0.0745	0.052	0.055	0.1360	0.095
^{234}U	0.0246	0.0673	0.047	0.042	0.1157	0.084
^{236}U	0.0400	0.0810	0.054	-	0.1393	0.097
^{238}U	0.0254	0.0727	0.050	0.071	0.1321	0.110
^{240}Pu	0.0158	0.0831	0.053	-	0.1420	0.101
^{242}Pu	0.0314	0.0807	0.054	-	0.1356	0.098
^{244}Pu	0.0390	-	0.061	-	-	0.109
^{246}Cm	0.0448	0.0929	0.055	-	0.1542	0.100
^{248}Cm	0.0360	0.1036	0.029	-	0.1742	0.106

1. Values are calculated from references 18, 20, and 21.
2. Input parameters are taken from Table 6.1.

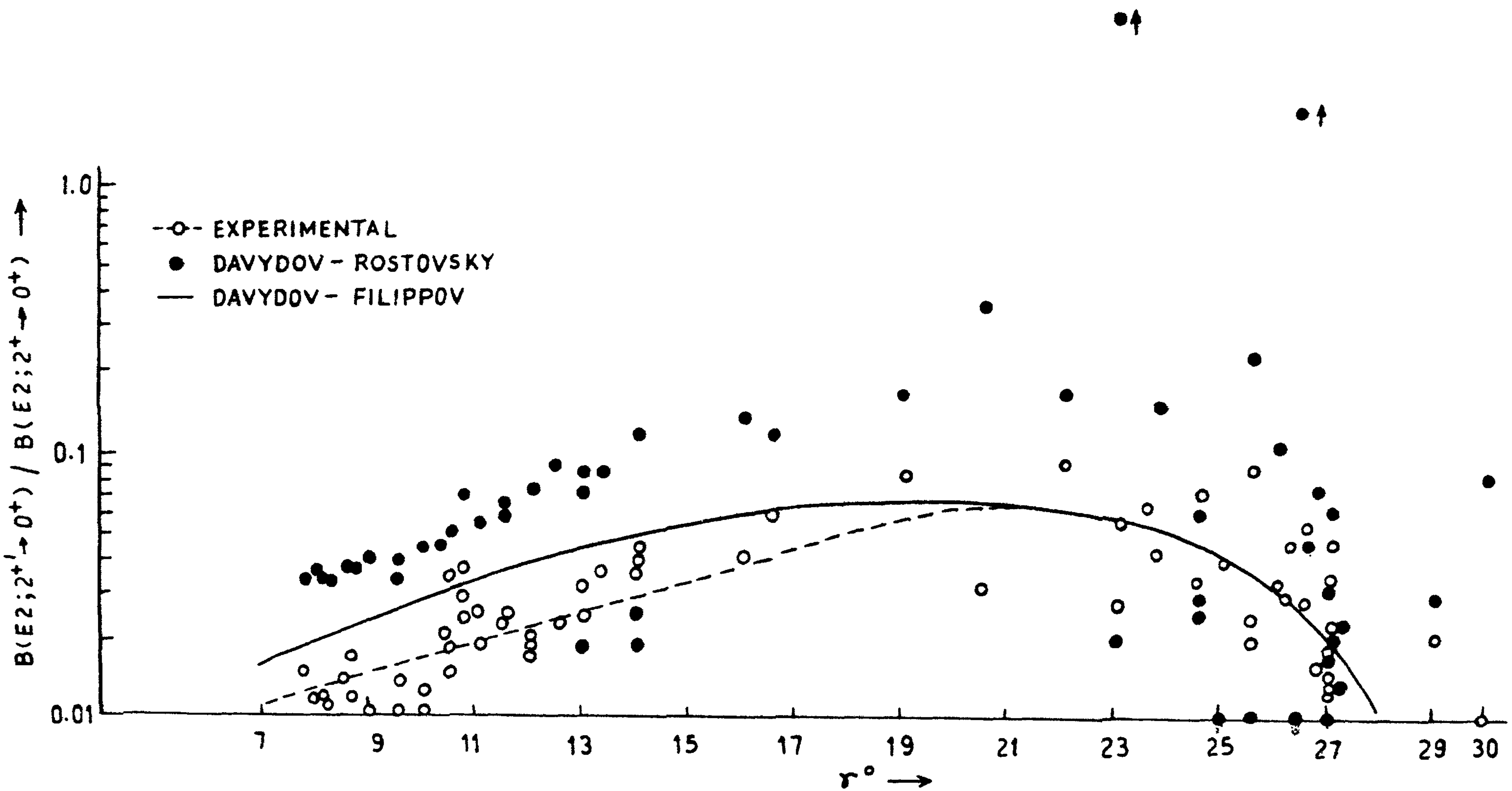

Fig. 6.1 Plot of $B(E2;2^{+'} \to 0^{+}) / B(E2;2^{+} \to 0^{+})$ values against non-axiality parameter γ

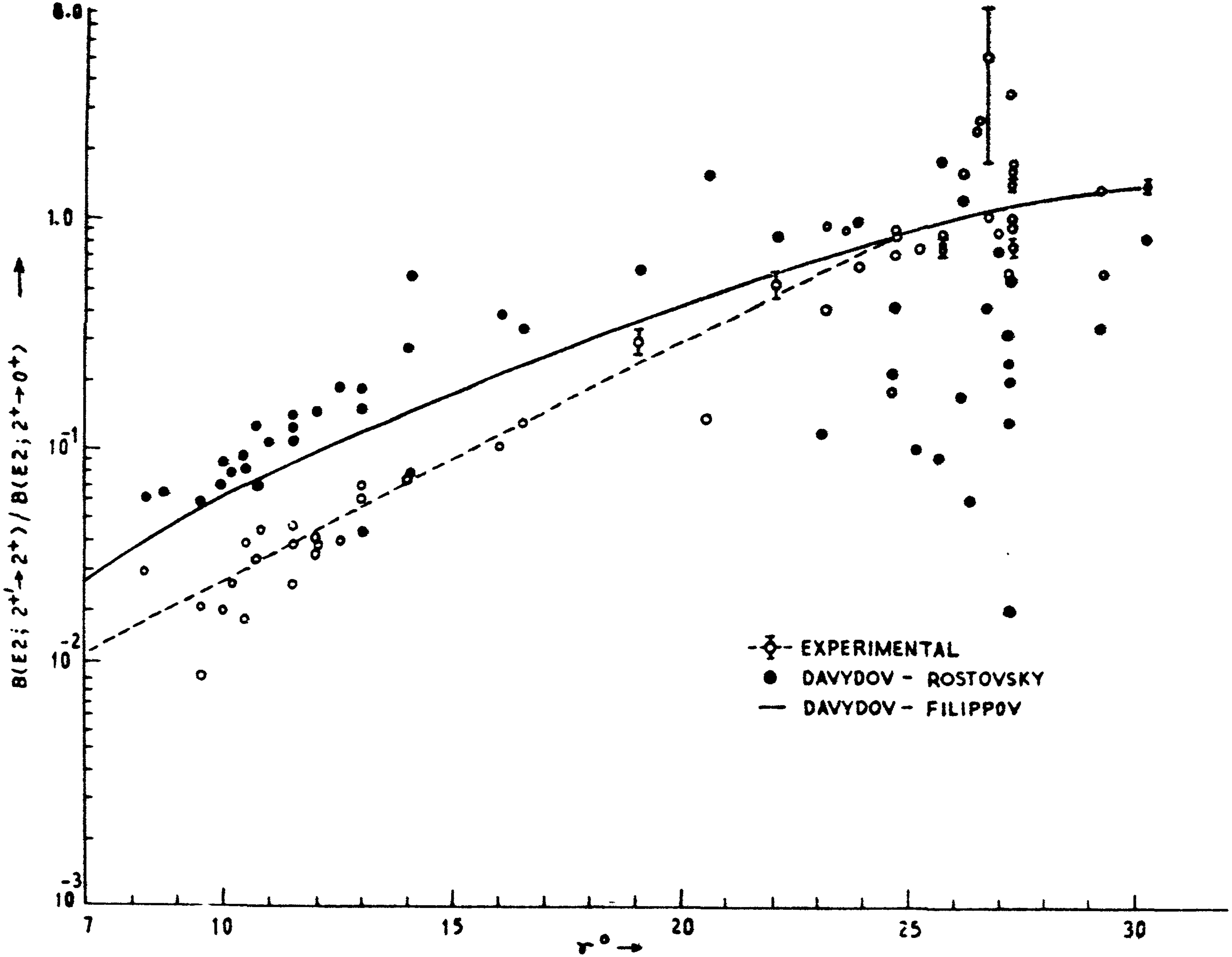

Fig. 6.2 Plot of $B(E2;2^{+'}\rightarrow 2^{+})/B(E2;2^{+}\rightarrow 0^{+})$ versus γ.

TABLE - 6.3

Predicted meanlives of $2^{+'}$ states as evaluated from Transition $2^{+'} \rightarrow 0^{+}$

Nuclues	γ (degree)	$B(E2: 2^{+} \rightarrow 0^{+})_{exp}$ $(e^{2}.b^{2})$	$2^{+'} \rightarrow 0^{+}$			
			$B(E2: 2^{+'} \rightarrow 0^{+})$ $(e^{2}.b^{2})$	E_{γ} (MeV)	I_{x}	τ (p.sec.)
^{108}Ru	22.5	0.185 (17)	0.0113 (10)	-	-	-
^{120}Te	30.0	0.105	$\approx$ 0	1.2011	0.17	-
^{130}Xe	26.0	0.198	0.0065	1.1222	0.136	0.6265
^{150}Nd	14.0	0.640	-	-	-	-
^{152}Gd	21.5	0.394	0.026	1.1092	0.46	0.8598
^{160}Gd	11.0	1.03	-	-	-	-
^{156}Er	23.5	0.33	0.0188	0.9305	0.28	1.742
^{158}Er	18.5	0.56	0.0316	0.8201	0.33	2.297
^{160}Er	15.84	0.84	0.0331	0.8544	0.39	2.112

Contd....

^{162}Er	13.3	0.976	-	-	-	-
^{160}Yb	(22.5)	0.464	0.0283	(0.712)	(0.43)	6.776
^{162}Yb	(17.0)	0.746 (112)	0.0324 (48)	(0.8749)	(0.43)	2.113 (273)
^{164}Yb	15.0	0.930	0.0319	0.8639	0.43	2.286
^{166}Yb	13.0	1.06	0.0276	0.9323	0.44	1.847
^{168}Yb	11.8	1.08	0.0237	0.9838	-	-
^{170}Yb	10.8	1.04	0.0208	1.1386	0.43	0.8815
^{174}Yb	9.0	1.15	0.0173	1.6341	-	-
^{176}Yb	10.0	1.05	0.012	1.261	0.44	0.942
^{166}Hf	(16.7)	0.86	0.0362	0.8101	0.54	3.489
^{168}Hf	(14.3)	0.836	0.0246	0.874	0.37	2.407
^{170}Hf	(13.0)	1.26 (27)	0.0328 (70)	0.9612	0.46	0.182 (32)
^{172}Hf	12.0	0.928	0.0203	1.0753	0.49	1.3701
^{180}W	14.0	0.863	0.0249	0.828	-	-
^{182}Os	14.8	0.702	0.0241	0.890	0.51	3.075
^{184}Os	14.0	0.569	0.0164	0.9429	0.47	3.138

Contd....

...

^{228}Th	9.8	1.41	0.0325	0.9688	0.37	1.505
^{232}U	9.4	1.98	0.0316	0.8669	0.43	2.267
^{236}U	8.7	2.32	-	-	-	-
^{238}Pu	8.3	2.52	0.0328	1.0285	0.41	0.8863
^{240}Pu	8.6	2.53	-	-	-	-
^{242}Pu	8.15	2.68	-	-	-	-
^{244}Pu	8.5	2.77	-	-	-	-
^{246}Cm	7.8	3.01	-	-	-	-
^{248}Cm	8.0	3.0	-	-	-	-

1. Values have been calculated using references 4, 18, 20 and 21.
2. Values of γ in brackets are deduced from Figure 5.7.
3. B(E2: $2^{+\prime} \rightarrow 0^{+}$) Values are obtained employing Figure 6.1.

TABLE - 6.4

Predicted Meanlives of $2^{+'}$ states as evaluated from Transition $2^{+'} \rightarrow 2^{+}$.

Nucleus	$2^{+'} \rightarrow 2^{+}$					Mean τ of $2^{+'}$ level (p.sec.)
	B(E2: $2^{+'} \rightarrow 2^{+}$) ($e^2.b^2$)	E_γ (MeV)	I_y	δ^2	τ (p.sec.)	
^{108}Ru	0.103 (9)	-	-	-	-	-
^{120}Te	0.1501	0.6408	0.83	53.29	4.176	4.176
^{130}Xe	0.2059	0.5861	0.86	53.29	4.928	-
^{150}Nd	0.0411	0.9323	-	201.64	-	-
^{152}Gd	0.1775	0.7649	0.51	156.25	0.8954	0.8776
^{160}Gd	0.0329	0.915	-	240.25	-	-
^{156}Er	0.2236	0.586	0.72	108.16	3.802	2.772
^{158}Er	0.1111	0.628	0.67	125.44	5.032	3.6675
^{160}Er	0.0778	0.7285	0.61	171.61	3.118	2.615

Contd....

^{162}Er	0.0509	0.7987	0.56	210.25	2.762	2.762
^{160}Yb	0.2591	(0.469)	(0.57)	75.69	7.911	7.3435
^{162}Yb	0.1051	(0.7086)	(0.57)	193.21	2.477	2.295
^{164}Yb	0.0702	0.7405	0.57	194.88	2.978	2.622
^{166}Yb	0.0541	0.8301	0.56	249.64	2.143	1.995
^{168}Yb	0.0491	0.8960	-	295.84	-	-
^{170}Yb	0.0332	1.0543	0.57	416.16	1.075	0.9782
^{174}Yb	-	1.5576	-	933.30	-	-
^{176}Yb	0.024	1.1787	0.56	542.89	0.8368	0.8894
^{166}Hf	0.1147	0.6514	0.46	161.29	2.790	3.1395
^{168}Hf	0.0516	0.750	0.63	219.03	2.790	3.3025
^{170}Hf	0.0671	0.8604	0.89	292.41	2.295	-
^{172}Hf	0.042	0.9800	0.51	384.16	1.096	1.2331
^{180}W	0.0554	0.724	-	237.16	-	-
^{182}Os	0.0530	0.7643	0.49	282.24	2.893	2.984
^{184}Os	0.0365	0.823	0.49	331.24	2.901	3.0195

Contd.....142....

^{228}Th	0.0323	0.9111	0.62	756.25	2.495	2.00
^{232}U	0.0427	0.8191	0.56	655.36	2.902	2.5845
^{236}U	0.0469	0.9126	0.73	829.44	2.006	2.006
^{238}Pu	0.0455	0.9844	0.57	1017.61	1.106	0.997
^{240}Pu	0.0512	0.8953	0.04	852.64	0.1108	0.1108
^{242}Pu	0.0413	1.0565	-	1204.09	-	-
^{244}Pu	0.0536	0.969	-	1022.72	-	-
^{246}Cm	0.0463	1.0815	0.54	1339.56	0.6435	0.6435
^{248}Cm	0.0462	1.007	-	1176.49	-	-

1. Values have been calculated from references 4, 18, 20 and 21.

2. B(E2: $2^{+\prime} \rightarrow 2^{+}$) Values are obtained employing Figure 6.2.

3. Mean τ is determined using Table 6.3, and is given in the last Column.

of τ calculated from predicted values of B(E2: $2^{+\prime} \rightarrow 0^{+}$) and B(E2: $2^{+\prime} \rightarrow 2^{+}$). The input parameters are also given in Tables 6.3 and 6.4.

6.3 Results and Discussion:

Fig.6.1 shows that at the beginning of region of deformation the experimental trend is lying lowest, DR trend has more deviation from experimental curve than DF trend. DF trend coincides with experimental one at $\gamma = 21^{\circ}$ with a gradual decrease in the deviation while there is no trend in DR values at the higher values of γ.

In Fig. 6.2 a similar situation is observed. DF trend is in the middle and there is gradual decrease in deviation from experimental one which ceases at $\gamma = 25^{\circ}$. There is no systematic trend in DR values in the range $20^{\circ} < \gamma < 30^{\circ}$.

Fig.6.3 shows an excellent agreement between DF and experimental values. This qualitative and quantitative reproduction of experimental findings has been found for all the γ-values and the mass regions of our interest. Many typical vibrational nuclei like ^{110}Pd, ^{122}Te in the region that brackets Z = 50 shall are found to obey DF discipline. B(E2: $2^{+\prime} \rightarrow 0^{+}$) values for ^{110}Pd which was too high in Weeks et al.[15] calculations, is also found to be reduced. B(E2: $2^{+\prime} \rightarrow 2^{+}/0^{+}$) values (which are more stringent test of a model) for ^{108,110}Pd nuclei lie within the experimental error limits in the present work.

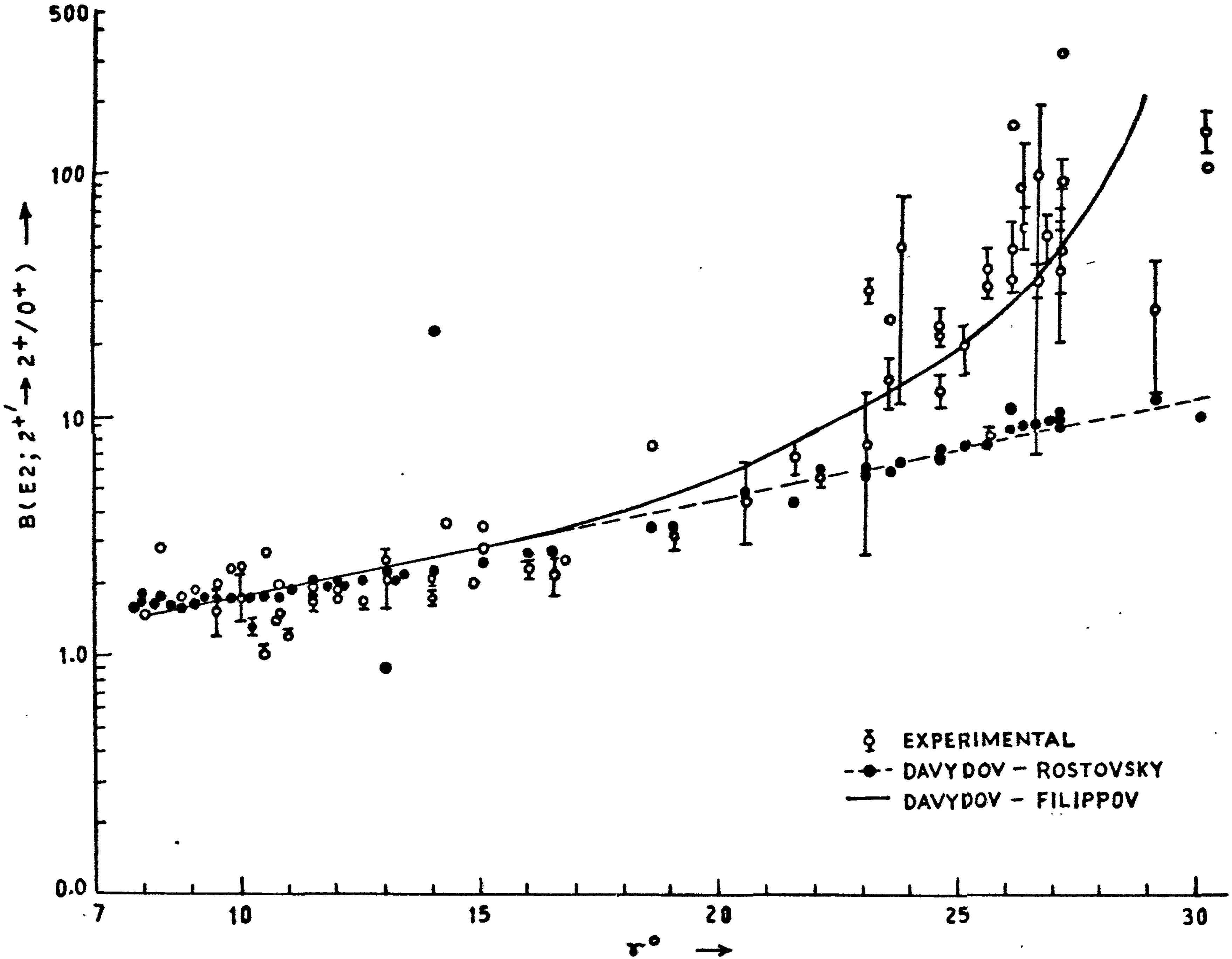

Fig. 6.3 Plot of $B(E2;\,2^{+\prime}\rightarrow 2^{+}/0^{+})$ versus γ.

The observed fine agreement between DF theory and experimental values for a large number of nuclei of known meanlives for $2^{+\prime}$ levels have encouraged us to predict B(E2) values and meanlives of 2^{+} state of gamma vibrational band for large number of cases [Tables 6.3 and 6.4].

6.4 Conclusion:

Since the branching ratio $2^{+\prime} \rightarrow 2^{+}/0^{+}$ offers an stringent test to judge a model, the inference drawn in present study lends support to rigid rotor model of Davydov and Filippov and the probable tri-axial shapes to nuclei in medium as well as heavy mass regions. The quantitative improvement over Hubel et al.[19)] analysis has further strengthened the concept of fixed β- and γ-parameters. It is also concluded that the assumption of rigid shape parameters is very useful approximation. The adiabatic approximation of DF model is quite reasonable if one is confined to low-lying excited states and to the probabilities for transitions between them. The couplings between rotation and β-vibration due to centrifugal forces are also ineffective at low spins. Therefore the assumption of rigidity with respect to change of the equilibrium value of parameter γ, which determines the deviation of nuclear shape from axial symmetry, should physically have no controversy.

The predicted values of meanlives of $2^{+\prime}$ states calculated from B(E2: $2^{+\prime} \rightarrow 0^{+}$) and B(E2: $2^{+\prime} \rightarrow 2^{+}$) are nearly same [Tables 6.3 and 6.4] except in nuclei ^{130}Xe and ^{170}Hf, The enhanced value of Υ , extracted from B(E2: $2^{+\prime} \rightarrow 2^{+}$) suggests that these transitions are not E2 dominated but may have M_1 component dominating in them. The discrepancy observed in our calculations is due to the DF values of δ, which probably is not reflecting the true mixing ratio in respect of ^{130}Xe and ^{170}Hf nuclei.

Our predictions of energies of $2^{+\prime}$ level in ^{166,170}Hf are 0.873 and 1.263 MeV and are much closer to their experimentally observed but suspected values of 0.81 and 1.214 MeV.

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NF 2

EFFECT OF SPIN ON NUCLEAR SHAPE IN EVEN-EVEN DEFORMED NUCLEI

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Abstract: The assumption of rigid triaxial shape with fixed shape parameters β and γ proves to be satisfactory as an approximation to the actual nuclear wave functions upto I=4. But at I =6, the nucleus deserves more freedom and the beta vibrations begin to influence the shape of the nucleus.

Transition probabilities from 2^+ to 0^+ and 4^+ to 2^+ states of ground band of even non-spherical nuclei using Davydov-Rostovsky model have been recently calculated by Agarwal/1/. There have been discrepancy in their results with experimental values, and they have scanned the nuclear regions of small γ values.

We are reporting Davydov et al/2/ calculations for $4^+ \to 2^+$ and $6^+ \to 4^+$ transitions using model dependent Q_0 after scanning large number of nuclei having large values of the non-axial parameter γ. Calculations reveal that DR model fails to explain almost all the medium mass nuclei and ^{148}Sm, ^{156}Er and ^{183}Pt nuclei possessing γ more than 20°.

Fig.1

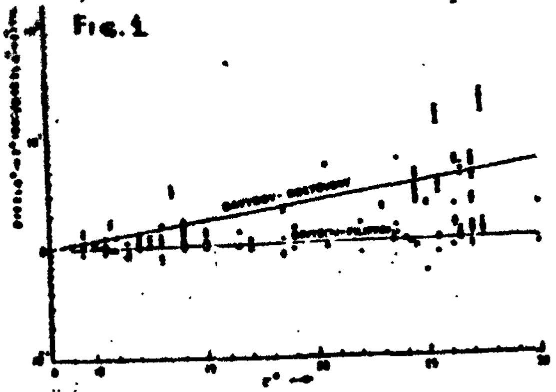

Fig.I: $B(E2;\ 4^+ \to 2^+)_{exp}/B(E2;4^+ \to 2^+)_{DF}$ and $B(E2;\ 4^+ \to 2^+)_{exp}/B(E2;\ 4^+ \to 2^+)_{DR}$ are plotted against γ.

A systematic in rigid rotor model values (Fig.1) has been obtained, while there is no systematic trend in DR model predictions for $4^+ \to 2^+$ transitions. The sudden appearance of systematic trend of DR model (Fig.2) for $6^+ \to 4^+$ transition suggests the begining of a process of shape transition in nucleus at I=6. This surprising change is observed in a number of nuclei on/near the trend for $6^+ \to 4^+$ transition in DF and DR predictions. The agreement is found to be reversed in both the models.

It is inferred that the assumption of rigid triaxial shape with fixed shape parameters β and γ are an excellent approximation to the actual nuclear wave functions upto spin 4 and thereafter it begins to get rid of rigid shape and acquires β and γ freedom. Ourstudy lends support to

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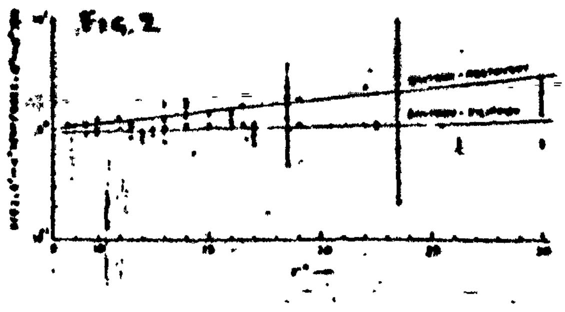

Fig.2: $B(E2;\ 6^+ \rightarrow 4^+)_{exp}/B(E2;\ 6^+ \rightarrow 4^+)_{DF}$ and $B(E2;\ 6^+ \rightarrow 4^+)_{exp}/B(E2;\ 6^+ \rightarrow 4^+)_{DR}$ are plotted against γ.

the assumption of triaxial shape of Davydov et al and suggests a limit at which the rigidity is unacceptable. This observation differs with Turner et al/3/ view point of abrupt phase transition of rotational mode from axially symmetric to asymmetric shape at I= 8.

The study of previous worker has been contradicted since the deformation parameter which they usedis subject to the following objection.

(1) Theoretical estimates for deformation parameter obtained by calculating the binding energies of the individual nucleus as a function of the nuclear deformation and minimizing the total energy have not been found consistent with the facts that nuclear deformation increase sharply as one moves away from closed shell regions, reflecting the polarizing effect of particles outside in some nuclei possessing mass number around 160/4/. Also there have been a discrepancy between experiment and theory connected with a fact that solution of BCS equation used are not corresponding to a definite number of particles in few cases near the begining of a region of deformation (viz around Hf and W isotopes)/4/. Although Agrawal used rather experimental values for deformation parameters but the data are now outdated and have changed to a considerable extent.

(2) Moreover the workers have scanned the nuclear regions which have small γ values and little transitional nuclei in the rare earth and actinide regions. In that case, the characteristic properties of asymmetric rotor model is not reflected on the systematics. Therefore the inference drown by above workers is not reliable. This is interesting to refer Hoehn et al/5/ who with the use of model dependent methods showed that electromagnetic moments involving the first excited state(2^+) and in some cases the second excited state (2^+) differ with those calculated values using pairing plus quadrupole interactions and others that has not been evident from previous measurements of lower precesion.

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NF3

LIFETIME PREDICTIONS FOR GAMMA VIBRATIONAL BAND IN EVEN EVEN DEFORMED NUCLEI

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Abstract: We have modified the figures of Gupta et al[Nuovo Cim.58B, 101(1980)] and included the medium mass nuclei for which $20^{\circ} < \gamma < 30^{\circ}$. On employing asymmetric rotor model dependent Q_0 it is observed that theoretical values of B(E2), branching ratios viz $2^{+\prime} \to 2^+/0^+$, $2^{+\prime} 2^+/2^+ \to 0^+$ and $2^{+\prime} 0^+/2^+ \to 0^+$ concide excellently with those of experimental one. The values of B(E2 ; $2^{+\prime} \to 0^+$), B(E2 ; $2^{+\prime} \to 2^+$) and mean lives of $2^{+\prime}$ state have been predicted, which are of much importance for the experimentalists.

There has been revival of interest/1/ in the asymmetric rotor model recently. Although the assumption of rigid tri-axial shapes with fixed parameters β and γ is concerned on an approximation to be actual nuclear wave functions but this has been turned out to be very useful, and is well supported by new data, most of them obtained from heavy ions experiments during the last decade. They give new support to the Davydov model and indicate that a number of transitional nuclei have triaxial shapes which are more stable that expected from theoretical potential energy surface calculation of Kumar/2/. Our previous work/3/ have already contradicted Zawischa's view point in general& confirmed that low lying $K^{\pi} = 2^+$ resonances are classical gamma vibrations. The qualitative and quantitative reproduction of experimental findings have been found irrespective of the value of γ and mass region. Many typical vibrational nuclei like ^{110}Pd, ^{122}Te in the region that bracketts Z=50 shell are found to obey DF discipline. The anomalous peak at ^{108}Pd while studying the mass number dependence of B(E2; $2^{+\prime} \to 2^+$) for Pd isotopes, which has been a long standing problem, has been brought in control in the present work. B(E2; $2^{+\prime} \to 0^+$) value for ^{110}Pd was too high in weeks calculating/4/ which also is found to be reduced. The general improvement over the recent calculations have encouraged us to predict B(E2)s and lifetime of $2^{+\prime}$ state of gamma vibrational band.

The adiabatic approximation is, thus, quite reasonable if one is confined to low lying excited states and the probabilities for transitions between them. The violation of the adiabatic condition during rotation, can be due to Coriolis interaction between the angular momentum and single nucleon states and to a centrifugal interaction which leads to a change of the shape of the nucleus during rotation and hence to a change of the moment of inertia.

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The predicted values of τ of $2^{+'}$ state are calculated from $B(E2;\ 2^{+'}\rightarrow 0^{+})$ and $B(E2;\ 2^{+'}\rightarrow 2^{+})$ transitions are similar except in nuclei ^{130}Xe and ^{170}Hf. The enhanced value of τ while extracting from $B(E2;\ 2^{+'}\rightarrow 2^{+})$ suggests that the transitions are not E2 dominated but the M1 component is larger in them. The discrepancy in our calculation is found due to employing DF value of δ, which probably is not reflecting on true mixing ratio in respect of ^{130}Xe and ^{170}Hf nuclei.

Table I: $B(E2;\ 2^{+'}\rightarrow 0^{+})$ and $B(E2;\ 2^{+'}\rightarrow 2^{+})$ are calculated and compared with their respective experimental values.

Nucleus	$B(E2;\ 2^{+'}\rightarrow 0^{+})$ Exp.	Cal.	$B(E2;\ 2^{+'}\rightarrow 2^{+})$ Exp.	Cal.
^{64}Ge	0.0013(5)	2.5×10^{-4}	0.038(14)	0.0877
^{76}Se	0.0015(3)	0.0025	0.087(3)	0.116
^{98}Mo	0.0027(4)	0.0018	0.166(23)	0.0621
^{100}Ru	0.0037(5)	0.0050	0.091(15)	0.088
^{102}Pd	0.0050(5)	0.0053	0.0055	0.0950
^{110}Pd	0.0026(2)	0.0037	0.180(30)	0.199
^{108}Cd	0.0058(7)	0.0041	0.078(9)	0.071
^{122}Te	0.0039(23)	0.0041	0.348	0.140

Table II : List of predicted values of $B(E2;\ 2^{+'}\rightarrow 0^{+})$ and meanlife of $2^{+'}$ state.

Nucleus	ζ	$B(E2;2^{+'}\rightarrow 0^{+})$ e^2b^2	E (MeV)	I	τ(ps)
^{130}Xe	25	0.0085	1.1222	0.133	0.6285
^{152}Gd	21.5	0.028	1.1092	0.46	0.86
^{156}Er	23.5	0.0188	0.9305	0.28	1.742
^{160}Yb	22.5	0.0235	0.712	0.43	0.776
^{166}Hf	16.7	0.0332	0.6101	0.54	3.5

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177211

Probable values of mean lives of rotational levels and B(E2) values of gamma-ray cascades

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Abstract. The systematic trend observed earlier between the ratio F $(=B(\mathrm{E2}: I+2 \rightarrow I)_{\mathrm{exp}}/B(\mathrm{E2}: I+2 \rightarrow I)_{\mathrm{theor}})$ and the non-axial parameter γ by Gupta *et al* may be of considerable interest to experimentalists. If one knows only γ and $B(\mathrm{E2}: 2^+ \rightarrow 0^+)$ of a deformed even nucleus, then using the systematics the absolute B(E2) values and probable mean lives of rotational levels can be predicted. Using the Davydov–Rostovsky model the $K^\pi = 0^+$ vibrational band-head energy (E_{0^+}) can also be calculated, with a controversial result. In the present paper the results of Gupta *et al* are modified by taking the asymmetric rotor-model-dependent intrinsic electric quadrupole moment (Q_0) instead of the value determined experimentally for the sake of consistency. Probable B(E2) values of gamma-ray cascades, mean lives of rotational levels and E_{0^+} have been predicted. Some of our predictions with respect to E_{0^+} are found to be in excellent agreement with their respective suspected values given in *Table of Isotopes*. The predicted mean lives are compared with their respective experimental values, if known, and are found to be in excellent agreement in almost all cases.

1. Introduction

The measurement of B(E2) values, mean lives and energies of excited states in nuclei is one of the most active areas of nuclear structure physics. During the last ten years it has been a major research point to devise new experimental techniques which can provide data giving unique and vital information, thus playing a major role in our understanding of nuclear structure. A knowledge of approximate values of mean lives is of great importance to experimentalists in selecting and setting up apparatus. There has also been a revival of interest in the asymmetric-rotor model recently (Meyer-ter-Vehn 1975, Meyer ter Vehn *et al* 1974, Lee *et al* 1977, Peker and Hamilton 1979, Turner and Kishimoto 1973, Bohr and Mottelson 1975, Tanabe and Sugawara 1976, Yamamura *et al* 1978, Marshalek 1975, Gupta *et al* 1980a, b). The reason for this increase in attention is partly the expectation that it may provide a reasonable phenomenological description of a nucleus in some domain of high angular momentum I. The striking success of the empirical relation (Gupta *et al* 1979, 1980a b) compared with other existing approaches in describing the B(E2) values of gamma ray cascades has contradicted the axial symmetry in the nucleus at lower spins (Turner and Kishimoto 1973). The assumption of a rigid triaxial shape with fixed shape parameters β and γ can be considered as an approximation to the actual nuclear wavefunctions, and this has been proved to be very useful and is well supported by new data, most of them obtained from heavy ion experiments during the last decade. The rigid

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nucleus begins to acquire some freedom at angular momentum $I=6$ and consequently obeys the Davydov–Rostovsky (1964: DR) model. This brings the $B(E2)_{DR}$ values of gamma ray cascades close to the systematic trend when plotted against the non-axial parameter γ. This significant change is useful in determining nuclear shape. These systematics are of immense importance to experimentalists since by using them the mean lives of rotational levels, absolute $B(E2)$ values of gamma-ray cascades and beta band-head energies can be predicted. A large number of nuclei with γ in the range $15° < \gamma < 25°$ and belonging to the medium mass region have been considered; these reflect the systematics of the asymmetric rotor model. It is remarkable that in this region there is no sound theoretical argument against the nucleus having a triaxial shape around $\gamma = 20°$. The error remaining in our previous calculations of total internal conversion coefficients with respect to uranium and thorium isotopes has also been removed.

Regarding the intrinsic electric quadrupole moment Q_0, which describes the deformation parameter β, we reject those theoretical values which have been widely used by previous workers (Rajput and Sehgal 1967, Rajput 1970, Augusthy *et al* 1972). This is because these theoretical estimates for the deformation parameter β, obtained by

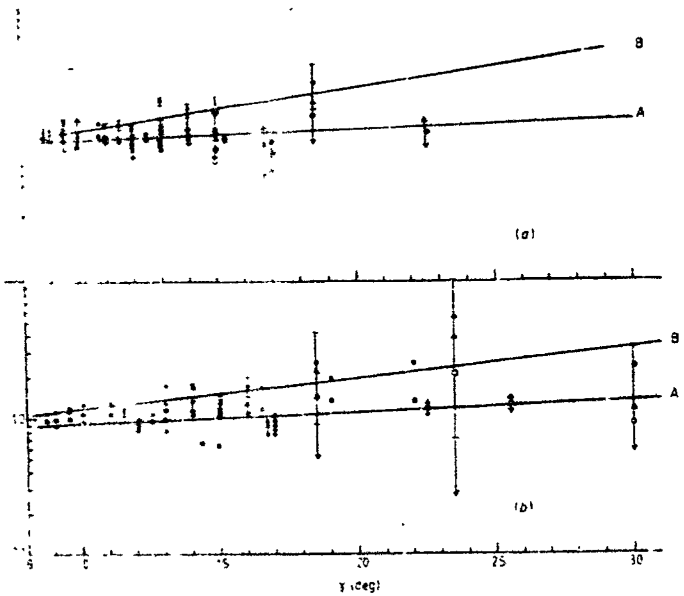

Figure 1. $F = B(E2: 6^+ \to 4^+)_{exp}/B(E2: 6^+ \to 4^+)_{theor}$ (b) and $F = B(E2: 8^+ \to 6^+)_{exp}/B(E2: 8^+ \to 6^+)_{theor}$ (a) are plotted against the non axial parameter γ expressed in degrees. O, A, empirical; ●, B, DR.

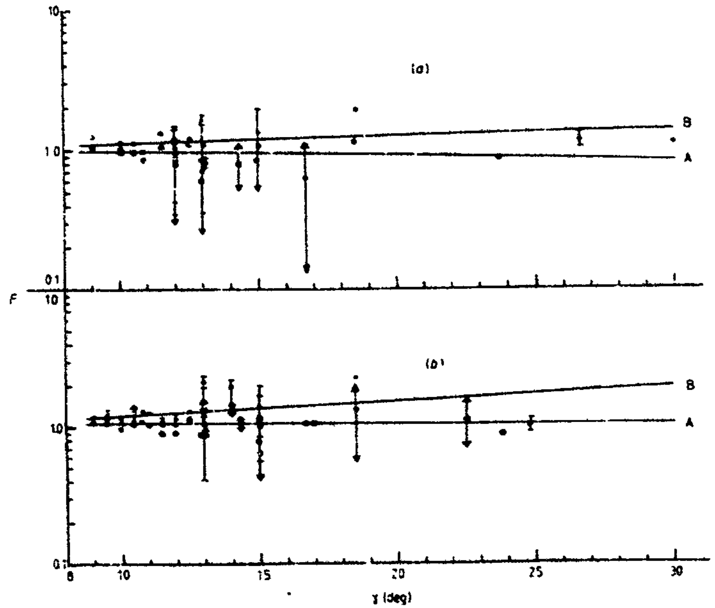

Figure 2. $F = B(E2: 10^+ \to 8^+)_{exp}/B(E2: 10^+ \to 8^+)_{theor}$ (b) and $F = B(E2: 12^+ \to 10^+)_{exp}/B(E2: 12^+ \to 10^+)_{theor}$ (a) are plotted against the non axial parameter γ expressed in degrees. O, A, empirical; ●, B, DR.

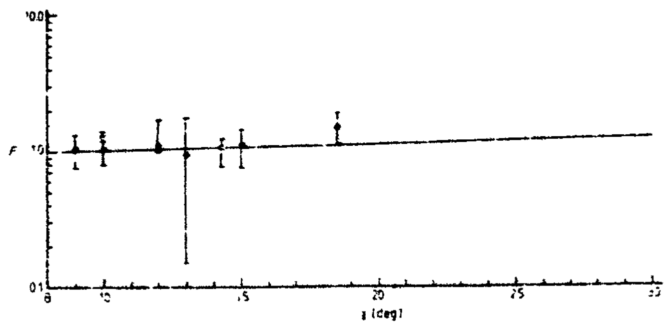

Figure 3. $F = B(E2: 14^+ \to 12^+)_{exp}/B(E2: 14^+ \to 12^+)_{DR}$ is plotted against the non axial parameter γ expressed in degrees.

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Table 1 Calculations of mean lives of rotational levels and $B(E2)$ values of gamma ray cascades The dependent parameter and its values are given in e^2 b^2 $B(E2)$ (in e^2 b^2), τ (in ps) and the energies of the rotational levels (in MeV) are given in various columns for the cascade $I \to 2 \to I$ with $I = 2$ 4 6 8 and 10 The bracketed values of γ are our predictions using the linear relationship of Gupta *et al* (1981) The values of γ are taken from the work of Davydov and Filippov $e^2 Q_0^2/16\pi$ is the asymmetric rotor model predicted values are given on line and their experimentally known values below the line with errors in parentheses

			$4^+ \to 2^+$			$6^+ \to 4^+$			$8^+ \to 6^+$			$10^+ \to 8^+$			$12^+ \to 10^+$		
Nucleus	γ (deg)	$e^2 Q_0^2/16\pi$ (e^2 b^2)	E_{4^+} (MeV)	$B(E2)$ (e^2 b^2)	τ (ps)	E_{6^+} (MeV)	$B(E2)$ (e^2 b^2)	τ (ps)	E_8 (MeV)	$B(E2)$ (e^2 b^2)	τ (ps)	E_{10} (MeV)	$B(E2)$ (e^2 b^2)	τ (ps)	E_{12} (MeV)	$B(E2)$ (e^2 b^2)	τ (ps)
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
^{92}Mo	18 5	0 218	0 743	0 296	15 41	1 305	0 342	4 25	1 957	0 404	1 71						
				0 253(42)	18 04(38)												
^{104}Mo	(14 0)	0 206	0.561	0 28	42.77	1 081	0 302	2 70									
^{106}Mo	(13 3)	0 251	0 5225	0 343	44 78	1 034	0 366	6 35									
^{98}Ru	27 0	0 0815	1 3978	0 128	2 76	2 2227	0 157	1 36	3 1267	0 185	0 73	4 001	0 139	1 15			
				0 108(12)‡	3 27(36)												
^{100}Ru	24 5	0 1077	1 2265	0 159	336	2 0777	0 194	9 41	3 062c	0 229	0 384	4 086	0 180	0 405			
				0 146(15)	366(36)												
^{102}Ru	25 5	0 1333	1 1064	0 200	40 6	1 875	0 247	1 23									
				0 201(38)	40 4(72)												
^{104}Ru	24 5	0 1659	0 888*	0 250	7 72												
				0 217(4)	8 66(60)												
^{108}Ru	22 5	0 185	0 6654	0 279	19 44												
^{110}Ru	(16 7)	0 1965	0 6639	0 269	22.37	1 24	0 30	4 28	1 9477	0 357	1 287						
^{112}Ru	(16 7)	0 227	0 6457	0 312	22 88												
^{104}Pd	23 0	0 095	1 2761	0 137	3 07	2 1112	0 165	1 22	3 0129	0 192	0 71	3 9926	0 16	0 56			
				0 147(9)‡	2.86(17)												
^{106}Pd	25 0	0 1082	1 3236	0 161	1 89	2.2495	0 198	6 06	3 2207	0 235	0 402	4 023	0 181	1 35			
				0 152(9)‡	2.00(12)												
^{106}Pd	26 5	0 1276	1 2292	0 197	2 25	2 0761	0 244	0 76	2 9625	0 287	0 52						
				0 22(3)	2 02(29)												
^{108}Pd	27 0	0.1494	1 0481	0 235	3 95	1 7713	0 291	1 44									
				0 28(4)	3 32(43)												
^{110}Pd	27 0	0 1746	0 9208	0 275	6 03	1 5739	0 339	2 02									
				0 31(4)	5.5(9)												
^{114}Pd	(21 6)	0 069	0 8536	0 098	21 75	1 5029	0 118	5 99									
^{116}Pd	(24 5)	0 116	0 8786	0 169	10 71												
^{108}Cd	24 5	0 0868	1 5085	0 127	1 26	2 5413	0 156	0 446	3 734	0 184	0 184						
				0 120(15)	1 33(19)												
^{110}Cd	26 0	0 1037	1 5424	0 130	1 16	2 4798	0 160	7 05	3 275	0 189	1 36	3 61	0 144	134 3			
^{112}Cd	27 0	0 1098	1 4142	0 173	1 46	2 167	0 214	1 56	2 8801	0 251	1 76						
				0 194(19)	1 29(14)												
^{114}Cd	27 0	0 1184	1 2833	0 186	2 18												
				0 212(36)†	1 92(32)												
^{116}Cd	25 5	0 1224	1 2195	0 184	2 52	2 0277	0 226	1 05									
				0 191(36)	2 45(57)												
^{120}Te	30 0	0 105	1 1613	0 172	6 05	1 7751	0 223	4 18	2 653	0 259	0 605						
^{122}Te	26 2	0 1359	1 1805	0 207	4 04	1 750	0 258	5 27	2 669	0 304	0 41	3 29	0 232	3 8			
				0.179†	5 09												
^{120}Xe	23 5	0 189	0 7946	0 275	12 5	1 3972	0 334	3 11	2 0996	0 391	1 22	2 871	0 315	0 94			
				0 85(32)	8 8(19)												
^{122}Xe	24 0	0 229	0 8288	0 335	7 97	1 4677	0 410	1 9	2 2184	0 482	0 71	3 036	0 382	0 58			
				0 325(51)	8 2(13)		0 19(3)	3 9(7)									
^{124}Xe	25 0	0 18	0 8787	0 266	7 71	1 5482	0.329	1 84	2 3307	0 391	0 71						
^{126}Xe	26 0	0 154	0 9416	0 234	6 72	1 6346	0 290	1 76	2 4335	0 343	0 72	3 315	0 26	0 59			
^{128}Xe	26 5	0 137	1 0327	0 209	5 47	1 737	0 262	1 78	2 513	0 308	0 94						

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Table 1 (continued)

Nucleus	γ (deg)	$e^2 Q_0^2$ 16π ($e^2 b^2$)	E_4 (MeV)	4⁺→2⁺ B(E2) ($e^2 b^2$)	4⁺→2⁺ τ (ps)	E_6 (MeV)	6⁺→4⁺ B(E2) ($e^2 b^2$)	6⁺→4⁺ τ (ps)
1	2	3	4	5	6	7	8	9
^{130}Ba	24.0	0.272	0.9017	0.359	4.75	1.5928	0.444	1.16
^{132}Ba	26.0	0.146	1.1276	0.218	2.92	1.9328	0.272	0.887
^{134}Ba	30.0	0.1413	1.4004	0.237	1.08			
				0.0204	12.5(26)			
^{136}Ba	30.0	0.082	1.866	0.137	0.47	2.2063	0.174	103.7
^{138}Ba	30.0	0.0426	1.8987	0.0705	54.25	2.0907	0.089	2916.5
^{132}Ce	24.5	0.329	0.8576	0.485	3.92	1.54	0.594	0.92
				0.437(109)	4.33(144)		0.476(35)	1.15(86)
^{134}Ce	25.5	0.214	1.0486	0.322	2.36	1.863	0.403	0.56
				0.170(27)	4.47(86)		0.075(32)	3.0(13)
^{140}Ce	30.0	0.036	2.0835	0.0604	48.8	2.1081	0.076	18.6×10^4
				0.0006	4978			10×10^6
^{142}Ce	25.0	0.0911	1.2193	0.135	9.31			(3×10^4)
^{144}Ce	(21.5)	0.173	0.6682	0.244	28.83	1.1711	0.296	8.5
^{148}Ce	(15.5)	0.368	0.4544	0.502	69.0	0.8409	0.552	17.1
^{150}Ce	(11.5)	0.755	0.308	1.19	136.8	0.5087	1.072	30.96
^{146}Nd	21.5	0.145	1.0434	0.208	5.51			
^{148}Nd	19.5	0.274	0.751	0.384	11.6	1.355	0.443	2.28
^{150}Nd	14.0	0.67	0.3825	0.911	87	0.7212	0.983	18.5
				0.501(71)	88(7)		1.00(7)	17.3(13)
^{154}Nd	(10.1)	1.012	0.2352	1.41	352.1	0.4789	1.426	53.86
^{148}Sm	(23.7)	0.1471	1.18	0.215	3.8	1.9061	0.261	1.55
				0.248(64)	3.32(86)			
^{150}Sm	20.5	0.2828	0.7733	0.391	13.52	1.2788	0.475	5.20
				0.521(39)	9.52(72)			
^{152}Sm	13.0	0.6863	0.3664	0.938	90.9	0.7072	1.00	17.6
				1.03(1)	82.8(11)		1.16(2)	14.7(4)
^{154}Sm	9.5	0.935	0.2669	1.30	226	0.5468	1.30	33.8
				1.18(3)	249(7)		1.31(4)	33.6(10)
^{152}Gd	21.5	0.394	0.755	0.557	12.5	1.2274	0.672	5.17
^{154}Gd	14.0	0.799	0.3712	1.086	71.8	0.7179	1.17	13.2
				1.18(4)	66.2(22)		1.36(5)	11.4(4)
^{156}Gd	11.0	0.9496	0.2882	1.32	160.3	0.5847	1.35	24.6
				1.29(2)	164(3)		1.47(3)	22.8(6)
^{158}Gd	10.5	0.978	0.2614	1.361	234	0.539	1.378	32.7
				1.37(11)	233(18)			
^{160}Gd	11.0	1.05	0.2482	1.45	272.8	0.514	1.479	37.8
^{156}Dy	15.5	0.751	0.40407	1.04	53.4	0.7705	1.131	10.92
^{158}Dy	13.0	1.013	0.3173	1.385	98.3	0.5378	1.474	16.34
				1.29(16)	108(13)			
^{160}Dy	12.0	1.019	0.2838	1.40	155.4	0.5812	1.46	23.9
				1.49(13)	146(13)		1.23(6)	28.4(11)
^{162}Dy	12.0	1.109	0.2657	1.52	191.2	0.5485	1.59	25.6
				1.53(9)	190(11)		1.38(16)	29.5(34)
^{164}Dy	12.0	1.1802	0.2423	1.622	237.1	0.5013	1.697	41.0
				1.57	245		1.67(6)	43.3(15)
^{156}Er	23.5	0.33	0.7972	0.48	8.9	1.3406	0.584	2.9
				0.54(7)	7.8(9)		1.02(89)	(6.9)

Nucleus	E_8 (MeV)	8⁺→6⁺ B(E2) ($e^2 b^2$)	8⁺→6⁺ τ (ps)	E_{10} (MeV)	10⁺→8⁺ B(E2) ($e^2 b^2$)	10⁺→8⁺ τ (ps)	E_{12} (MeV)	12⁺→10⁺ B(E2) ($e^2 b^2$)	12⁺→10⁺ τ (ps)
1	10	11	12	13	14	15	16	17	18
^{130}Ba	2.396	0.522	0.47	3.365	0.414	0.23			
^{132}Ba	2.796	0.321	0.53	3.115	0.244	106.1			
^{132}Ce	2.3268	0.719	0.37	3.1545	0.563	0.37			
^{134}Ce	2.8117	0.471	0.226	3.7295	0.36	0.36			
					0.015(2)	8.6(14)			
^{150}Ce	0.9851	1.289	8.38						
^{150}Nd	1.1307	1.17	6.03						
		1.02(9)	6.8(6)						
^{154}Nd	0.807	1.702	12.6						
^{148}Sm	2.5433	0.313	2.48	3.16	0.239	3.89			
^{150}Sm	1.9371	0.564	2.76	2.4328	0.487	2.23	3.048	0.406	2.28
^{152}Sm	1.1254	1.19	5.3	1.608	1.78	1.74	2.158	0.90	1.79
		1.41(7)	4.4(2)		1.54(14)	1.99(18)			
^{154}Sm	0.903	1.56	9.1						
		1.54(15)	8.9(8)						
^{152}Gd	1.747	0.76	2.83	2.3	0.648	2.43	2.884	0.548	2.19
^{154}Gd	1.1447	1.40	4.1						
		1.50(5)	3.7(1)						
^{156}Gd	0.9652	1.65	6.1						
		1.61(11)	6.2(4)						
^{158}Gd	0.9044	1.66	7.51	1.3505	1.73	2.66	1.8667	1.50	1.40
		1.69(13)	7.36(57)		1.77(9)	2.60(14)		1.57(12)	1.41(11)
^{160}Gd	0.868	1.789	8.20						
^{156}Dy	1.2158	1.349	3.45	1.7249	1.28	1.85	2.285	1.12	1.31
^{158}Dy	1.0441	1.755	4.19	1.5195	1.75	1.91			
^{160}Dy	0.9672	1.74	5.4	1.4286	1.77	2.2	1.9517	1.6	1.3
		1.57(10)	4.9(4)		1.65(15)	2.3(2)		1.29(79)	1.6(2)
^{162}Dy	0.9213	1.89	5.96	1.3751	1.93	2.2	1.901	1.74	1.16
		1.91(8)	5.90(50)		1.79(1)	2.3(2)		1.46(8)	1.3(7)
^{164}Dy	0.8437	2.02	8.5	1.2613	2.05	3.12	1.7453	1.96	1.64
		1.69(12)	9.8(2)		1.89(10)	3.4(1)		1.87(3)	1.5(1)
^{156}Er	1.9589	0.67	1.34	2.633	0.55	0.94	3.315	0.46	1.2

Table 1 (continued)

			$4^+ \to 2^+$			$6^+ \to 4^+$			$8^+ \to 6^+$			$10^+ \to 8^+$			$12^+ \to 10^+$		
Nucleus	(deg)	$e^2Q_0/16\pi$ ($e^2 b^2$)	E_4 (MeV)	$B(E2)$ ($e^2 b^2$)	τ (ps)	E_6 (MeV)	$B(E2)$ ($e^2 b^2$)	τ (ps)	E_8 (MeV)	$B(E2)$ ($e^2 b^2$)	τ (ps)	E_{10} (MeV)	$B(E2)$ ($e^2 b^2$)	τ (ps)	E_{12} (MeV)	$B(E2)$ ($e^2 b^2$)	τ (ps)
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
^{158}Er	18 5	0 56	0 5272	0 76	24 3	0 9703	0 882	2 6	1 4932	1 04	2 0	2.072	0 93	1 34	2 68	0 80	1 22
				0 88(4)	20 8(10)		1 15(72)	4 0(7)		1 28(45)	1 6(5)		1 13(66)	1 1(5)		0 96	1 0
^{160}Er	15 0	0 84	0 3895	1 14	51 1	0 7648	1 25	8 7	1 2285	1 49	2.5	1 7606	1 43	1 33	2 3395	1 26	0 99
				1 17(6)	49 8(24)		1 36(12)	7 8(7)		1 17(28)	3 2(7)		1 09(55)	1 7(7)			
^{162}Er	13 3	1 013	0 3295	1 33	84 4	0 6670	1 433	13 0	1 0968	1 737	3 2						
^{164}Er	13 0	1 186	0 2995	1 62	108	0 6144	1 727	4 72	1 0246	2 06	3 39	1 5181	2 05	1 35	2 0827	1 84	0 77
				1 40(13)	124(11)					1 83(13)	3 82(27)		2 97(12)	1 41(9)		1 20(9)	1 18(9)
^{166}Er	12.5	1 148	0 265	1 57	181 7	0 5454	1 66	26 8	0 9112	1 98	6 3	1 3497	1 99	2 5	1 8472	1 79	0 67
				1 69(11)	168 8(101)		1 60	29 6		1 85(11)	6 8(4)		2 1(2)	2 3(2)		1 97(15)	1 3(1)
^{168}Er	12.0	1 195	0 2641	1 64	178	0 5487	1 719	23 54	0 9282	2 04	5 06						
				1 67(10)	173(12)					2 10(18)	4 91(43)						
^{170}Er	11 5	1 203	0 2601	1 66	191	0 5411	1 711	25 20	0 913	2 06	5 54	1 374	2 11	1 85	1 916	1 92	0 90
				1 55(4)	204(5)					2 21(16)	5 16(37)		1 83(12)	2 13(14)		2 12(10)	0 82(4)
^{162}Yb	17 0	0 746	0 4866	1 01	23 8	0 9233	1 148	4 4	1 4447	1 36	1 55	2 0233	1 247	1 01	2 6338	1 088	0 88
				1 19(17)	20 2(28)		1 10(33)	4 6(9)		1 04(4)	2 02(7)						
^{164}Yb	15 0	0 93	0 3857	1 264	46 4	0 7604	1 39	7 9	1.2234	1 65	2.3	1 754	1 58	1 22	2 3307	1 39	0 92
				1 37(5)	42 9(16)		1 47(5)	7 2(3)		1 73(65)	2 2(7)		1 61(62)	1 2(4)		1 59(71)	0 79(30)
^{166}Yb	13 0	1 06	0 3305	1 45	77 4	0 668	1 55	11 9	1 0983	1 84	3 0	1 6059	1 84	1.31	2 1756	1 64	1 2
				1 47(5)	76 3(24)		1 57(6)	11 2(4)		1 75(2)	3 1(3)		1 65(100)	1 4(7)		1 46(102)	0 92(47)
^{168}Yb	11 8	1 08	0 2865	1 50	136 2	0 5853	1 559	20 77	0 97	1 865	5 19	1 4254	1 894	2.20	1 936	1 718	1 37
^{170}Yb	10 8	1 04	0 2774	1 44	163 8	0 5731	1 475	23 12	0 963	1 79	4 75	1 4371	1 84	1 84	1 983	1 68	1 00
										1 61(14)	4 28(35)		2 02(13)	1 67(11)		1 51(12)	1 11(9)
^{166}Hf	(16 7)	0 86	0 47	1 165	23 0	0 897	1 31	3 4	1 406	1 568	1 5	1 970	1 44	0 99	2 564	1 26	0 87
				1 10(7)	24 2(16)		1 14	5 0(7)		1 38(00)	1 7(7)		1 42(00)	1 0(7)		0 84(64)	1 3(10)
^{168}Hf	(14 3)	0 836	0 3853	1 138	52 0	0 7565	1 24	8 8	1 2[illegible]3	1 47	2 8	1 735	1 42	1 47	2.305	1 27	0 94
				1 14(13)	51 9(58)		1 28(13)	8 5(9)		1 42(00)	2 9(3)		1 50(00)	1 4(2)		1 59(4)	0 75(2)
^{172}Hf	12 09	0 928	0 3093	1 27	54 07	0 6281	1 33	18 63	1 0342	1 577	4 63	1 521	1 616	1 905	2 0645	1 452	1 18
^{174}Hf	10 8	0 9626	0 2974	1 33	125 99	0 6084	1 367	20 55	1 0094	1 629	4 83	1 4856	1 698	1 962	2 02	1 536	1 22
^{176}Hf	10 25	1 235	0 2902	1 71	109 68	0 597	1 739	17 26	0 998	2 089	3 77	1 4813	2.182	1 418	2 0349	2 008	0 78
^{178}Hf	11 5	1 021	0 3066	1 40	102 89	0 5322	1 448	15 42	1 0585	1 731	3 34	1 571	1 776	1 301	2 1507	1 614	0 77
^{180}Hf	10 7	0 974	0 3086	1 355	108 2	0 6408	1 785	14 54	1 0839	1 661	2 88						
				1 32(17)	115 4(72)												
^{180}W	14 0	0 863	0 3379	1 18	19 28	0 6888	1 264	12.13	1 1391	1 512	2.91	1 665	1 473	1 378	2 237	1 316	1 01
^{182}W	11 5	1 238	0 3294	1 714	66 4	0 6805	1 77	8 6	1 1445	1 806	2 10	1 7121	1 851	0 748	2.373	1 677	0 386100
				1 235(90)	92.3(72)		1 22(13)	12 5(13)									
^{184}W	14 0	0 695	0 364	0 9467	68 4	0 7483	1 02	9 [illegible]									
				0 954(9)	67 8(58)		1 24(13)	[illegible] 8(8)									
^{186}W	16 0	0 697	0 3965	0 945	49 0	0 8085	1 05	6 5									
				0 892	51 9		1 36	5 0									
^{182}Os	14 8	0 702	0 4002	0 97	48 61	0 7932	1 045	8 28	1 2773	1 237	2 48	1 811	1 187	1.586	2.345	1 053	1 78
^{184}Os	14 0	0 560	0 3838	0 774	74 4	0 7741	0 833	10 80	1 2749	0 997	2 60	1 870	0 965	1 133	2 546	0 845	0 668
				0 84(26)	66 4(187)												
^{186}Os	16 5	0 667	0 4341	0 904	37 0	0 8689	1 02	5 1	1 4209	1.217	1 31	2.069	1 130	0 632	2 782	0 982	0 45
				0 90	34 6		1 16	4 5									
^{188}Os	19 0	0 617	0 478	0 841	27 5	0 9398	0 98	3 9	1 5147	1 026	1 28	2 170	2.079	0 325			
				0 79	29 2												
^{190}Os	22 0	0 543	0 5478	0 774	17 1	1 0503	0 94	2 69	1 6664	1 058	0.87						
				0 62	20 2		1 05	2 45									
^{192}Os	25 [illegible]	0 459	0 5803	0 69	16 0	1 0886	0 85	2 81	1 7081	1 00	0 89						
				0 54(6)	20 2(22)		0 87(14)	2.74(43)									

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Table 1. (continued)

Nucleus	γ (deg)	$e^2Q_0^2/16\pi$ (e^2 b^2)	4$^+$→2$^+$ E_{4^+} (MeV)	B(E2) (e^2 b^2)	τ (ps)	6$^+$→4$^+$ E_{6^+} (MeV)	B(E2) (e^2 b^2)	τ (ps)
1	2	3	4	5	6	7	8	9
^{184}Pt	19.5	1.07	0.4365	1.48	31.98	0.7981	1.716	7.69
^{186}Pt	21.5	0.71	0.4901	0.987	31.66	0.8772	1.206	7.78
^{188}Pt	26.5	0.52	0.6706	0.82	9.13	1.1846	1.00	2.27
^{190}Pt	28.5	0.52	0.7369	0.83	5.88	1.2875	1.058	1.52
^{228}Th	9.8	1.41	0.1869	1.986	203.0	0.3782	1.967	107.94
				1.72(7)	232.3(72)			
^{230}Th	10.5	1.625	0.1737	2.26	243.0	0.357	2.285	115.55
				2.29(7)	239.5(72)			
^{232}[illegible]	9.4	1.98	0.1566	2.76	183.88	0.3216	2.757	91.15
^{234}[illegible]	8.7	2.053	0.1433	2.83	278.70	0.2961	2.783	130.80
[illegible]	8.3	2.395	0.1484	3.33	197.44	0.3072	3.239	93.96
^{238}[illegible]	8.3	2.52	0.1460	3.51	172.65	0.3036	3.425	84.50
[illegible]	8.6	2.536	0.1417	3.53	135.72	0.2943	3.453	95.19
[illegible]Pu	8.15	2.711	0.1495	3.76	145.61	0.3097	3.643	75.82
[illegible]Pu	8.5	2.799	0.154	3.87	117.15	0.315	3.791	71.06
^{248}Cm	7.8	3.045	0.1420	4.21	102.09	0.2956	4.083	77.92

Nucleus	8$^+$→6$^+$ E_8 (MeV)	B(E2) (e^2 b^2)	τ (ps)	10$^+$→8$^+$ E_{10} (MeV)	B(E2) (e^2 b^2)	τ (ps)	E_{12} (MeV)	B(E2) (e^2 b^2)	τ (ps)
1	10	11	12	13	14	15	16	17	18
^{184}Pt	1.2310	1.989	2.73	1.7074	1.753	1.89	2.205	1.492	1.79
^{186}Pt	1.3411	1.368	2.78	1.857	1.165	1.93	2.335	0.983	3.33
^{188}Pt	1.7823	1.18	0.906	2.438	0.894	0.76	2.676	0.724	29.24
^{190}Pt	1.9151	1.25	0.670	2.5353	0.906	0.98	2.7267	0.739	335.2
^{228}Th									
^{230}Th	0.592	2.735	30.27						
^{232}[illegible]	0.5406	3.307	31.20	0.805	3.521	15.71	1.112	[illegible]	8.55
^{234}[illegible]	0.4971	3.40	46.7	0.741	3.618	20.86	1.024	3.358	11.95
[illegible]	0.5183	3.878	32.23	0.7757	4.255	14.01	1.0765	3.972	[illegible]
^{238}[illegible]	0.514	4.09	29.78						
[illegible]	0.4976	4.131	34.47	0.751	4.514	13.52			
[illegible]Pu	0.5227	4.273	26.16	0.779	4.806	11.04	1.087	4.529	6.50
[illegible]Pu	0.537	4.527	20.89						
^{248}Cm	0.5004	4.86	27.25						

† [illegible]moto and Tamura (1976)
‡ Weeks and Tamura (1980)

calculating the binding energies of the individual nuclei as a function of the nuclear deformation and minimising the total energy, are not consistent with the fact that the nuclear deformation increases sharply as one moves away from closed shell regions. This sharp increase in nuclear deformation reflects the polarising effect of particles outside closed shells in nuclei with mass number around 160 (Bes and Szymanski 1961, Szymanski 1961). There has also been a discrepancy between experiment and theory connected with the fact that solutions of the BCS equation used do not correspond to a definite number of particles near the beginning of the deformation region namely in Hf and W isotopes. Agarwal (1978) did use experimental values of the deformation parameters but the data of Stelson and Grodzins (1965) used by him are now outdated and have changed to a considerable extent. In addition, his work was limited to the nuclear region which has small γ values and he considered only a small number of transitional nuclei in the rare earth and actinide regions. In Agarwal's case the characteristic properties of the asymmetric rotor model (ARM) were not reflected in the systematics. We use for the first time the ARM dependent Q_0 given in terms of easily observable transition probabilities $B(E2;\ I_i \to I_f)$ by

$$e^2Q_0^2 = 16\pi(B(E2;\ 2^+ \to 0^+) + B(E2;\ 2^{+\prime} \to 0^+))$$

to keep consistency in the theoretical predictions (Alder and Steffen 1964).

We have modified the results of Gupta et al (1980a, b) [illegible]ving model dependent Q_0 and have extended their results up to $12^+ \to 10^+$ transit[illegible] [illegible] have predicted absolute $B(E2)$ values, mean lives of excited rotational levels and be[illegible] head energies E_0.

2. Method of calculation

$B(E2)_{exp}$ values have been extracted from mean lives using known relations (Ross and Bhaduri 1972). E_i the excitation energy, τ the mean life of the excited state and α_T, the total internal conversion coefficient, have been taken from Lederer and Shirley (1978). The parameters s, g and γ have been calculated from the experimentally known energy levels E_{2^+}, $E_{2^{+\prime}}$ and E_0 compiled by Sakai and Rester (1977). The empirical values of $B(E2)$ have been calculated using the relation of Gupta et al (1980a, b) while the $B(E2)_{DR}$ values are extracted from the model relations of Davydov and Rostovsky (1964).

2.1 Probable B(E2) values and mean lives

The ratio $F_I = B(E2;\ I+2 \to I)_{exp}/B(E2;\ I+2 \to I)_{theor}$ has been plotted against the non axial parameter γ for $I = 4$, 6, 8 and 10 (figures 1–3). For a particular nucleus the non axial parameter γ is determined from the ratio $E_{2^+}/E_{2^{+\prime}}$ (taken from Sakai and Rester 1977) and $B(E2)_{exp}$ is estimated using the equation of Gupta et al (1980a, b) leading to an ARM dependent Q_0. The results plotted yield F for a particular γ and this in turn gives $B(E2)_{calc}$. This value is approximate and can be calculated for those levels whose mean lives are as yet unknown. The mean life can be computed using the relation

$$B(E2;\ I+2 \to I)_{calc} = \frac{0.0812}{(1+\alpha_T)\tau E_\gamma^5}$$

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where B(E2) is in e^2 b^2. E_γ is in MeV and τ is in ps. The probable values of the mean lives τ and the B(E2) values of gamma-ray cascades are displayed in table 1. We have used the energies and mean lives of the 2^+ and $2^{+\prime}$ states of the rotational band to calculate Q_0 and the mean lives of higher angular momentum states of the rotational band have been predicted.

2.2. Beta band-head energy

A remarkable systematic trend observed in F as a function of γ using the Davydov–Rostovsky model calculations for $I > 4$ (see figures 1–3) has stimulated us to reval the unknown beta band-head energies. For a nucleus whose $2^{+\prime}$ level is unknown, we have used a linear relationship (Gupta *et al* 1981) to predict the approximate value of γ feeding $E_{2^{+\prime}}$ and A. This γ in turn gives us the appropriate factor F using figures 1, 2 and 3. Knowing $B(\mathrm{E2}; I+2 \to I)_{\mathrm{exp}}$, we can determine $B(\mathrm{E2}; I+2 \to I)_{\mathrm{DR}}$. These values are reduced to give $B(\mathrm{E2}; 2^+ \to 0^+)_{\mathrm{DR}}$ and the Clebsch–Gordan coefficients. On substituting s and Q_0, the value of q can be extracted since $q\, E_{2^{+\prime}} = E_{0^{+\prime}}$. In this way we can calculate the beta band-head energy ($E_{0^{+\prime}}$). Values of $B(\mathrm{E2}; 2^+ \to 0^+)_{\mathrm{DR}}$, $E_{2^{+\prime}}$, $e^2Q_0^2/16\pi$, s, q, $E_{0^{+\prime}}$ and the suspected $E_{0^{+\prime}}$ are listed in table 2.

It is surprising to note that, in the nuclei ^{176}Yb, ^{166}Hf and ^{168}Hf, the energy levels already observed experimentally (at 1.34, 0.69 and 0.94 MeV) but of doubtful spin assignment are very close to our empirical results (1.36, 0.72 and 0.91 MeV). This striking agreement within an error of 5% confirms the experimentally observed but suspected levels as $0^{+\prime}$ for those nuclei. Although a present we have considered only a few nuclei, our method can be extended successfully to other nuclei in the heavy-mass deformed nuclear region.

3. Conclusion

With the development of new experimental techniques, high-energy heavy ions carry large angular momentum, which the compound nucleus cannot get rid of by evaporation of neutrons. Therefore, high-spin states result in the final nucleus and the corresponding gamma-ray cascades have been observed. In even–even nuclei several cascades, interpreted as coming from 30^+, states have been identified. We notice that the energy spacing between the higher levels in different nuclei is almost constant for the same value of I varying from 8 to 16. This indicates that the moments of inertia of the rotating nuclei in these high-spin states become almost the same. The centrifugal stretching increases the deformation, pairing becomes less important and the nucleus behaves as a rigid rotor. Retaining an almost spheroidal shape but allowing stretching leaves only one parameter, γ, free to describe the change in the moment of inertia in going up the band. Extremely good agreement has been obtained with variation of this one parameter. This has obviously encouraged us to predict B(E2) and τ using a one-parameter-dependent empirical relation. Although there is a risk of not getting rotational bands at high-spin states since, if more and more pairs are broken up, the wavefunction of the higher-spin states no longer resembles that of the ground state and the rotational band breaks off (Mottelson *et al* 1960). So far there is no such example of rotational band break-off, although the limit predicted by Mottelson *et al*, namely $I = 14$–18, has already been passed in some cases. For lighter nuclei such as Xe the limit lies at spin 10 or 12.

We have compared our predictions of B(E2) values and mean lives with their respective

Table 2. Calculations of $K^\pi = 0^+$ and 2^+ vibrational band head energies of collective excitations. In columns (a), (b), (c), (d), (e) and (f) the $B(\mathrm{E2}; 2^+ \to 0^+)_{\mathrm{DR}}$ are displayed using figures 1(a, b), 2(a, b) and 3 and their mean values respectively. $B(\mathrm{E2}; 2^+ \to 0^+)_{\mathrm{exp}}$ values are taken from Lederer and Shirley (1978). $E_{2^{+\prime}}$ is the energy of the $K^\pi = 2^+$ vibrational band head given by Sakai and Rester (1977). The bracketed values are our predictions using the linear relationship of Gupta *et al* (1981). s and q are parameters calculated using the table of Sakai and Rester (1977). $E_{0^{+\prime}}$ and $E_{0^+_T}$ are the predicted and suspected energies of the $K^\pi = 0^+$ vibrational band head adapted from Lederer and Shirley (1978) and expressed in keV.

Nucleus	$B(\mathrm{E2}; 2^+ \to 0^+)_{\mathrm{DR}}$ (e^2 b^2) (a)	(b)	(c)	(d)	(e)	(f)	$e^2Q_0^2/16\pi$ (e^2 b^2)	$E_{2^{+\prime}}$ (keV)	s	q	$E_{0^{+\prime}}$ (keV)	$E_{0^+_T}$ (keV)
^{164}Dy	0.814(12)	0.735(48)	0.869(6)	0.987(17)	0.964(52)	0.874(31)	1.14	761.8	10.38	(9.36)	(687)	—
^{160}Yb	0.248(25)	0.172(43)	0.465(128)	—	—	0.295(65)	0.462(115)	(713)	(2.93)	(11.05)	(2685)	—
^{162}Yb	0.407(76)	0.310(79)	—	—	—	0.358(77)	0.746(112)	(875)	(5.26)	(3.90)	(648.3)	—
^{176}Yb	0.807(102)	—	0.940(77)	0.952(87)	0.947(178)	0.911(111)	1.05	1261	15.35	(16.63)	(1365.8)	1340
^{192}Os	0.210(32)		—	—	—	0.210(32)	0.420	489.0	2.38	(4.69)	(965)	956
^{166}Hf	0.432(63)	0.401(88)	0.591	—		0.475(125)	0.86	(869)	(5.47)	(4.53)	(718.9)	695
^{168}Hf	0.547(57)	0.510(48)	0.638(94)	—	0.694(153)	0.597(88)	0.836	(940)	(7.60)	(7.38)	(912.9)	940